Abstract
We investigate the effect of employee heterogeneity on the incentive to exert effort in a market-based tournament. External employers use promotion decisions to estimate employees’ abilities and adjust their wage offers accordingly. Employees exert effort to increase the probability of being promoted and thus to increase their ability assessment and wage offer. We demonstrate that ability assessments and wage offers are more sensitive to promotion decisions in the case of heterogeneous employees. Thus, employees have a higher incentive to affect the tournament outcome, and employers find it optimal to hire heterogeneous employees.

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1 Introduction

In many firms, promotions form an important incentive. Employees exert effort to perform better than their colleagues and, thus, to be considered for promotion and concomitantly offered an increase in compensation. The current paper analyzes how heterogeneity among employees affects the incentive to exert effort in a promotion tournament. A rationale for designing heterogeneous promotion tournaments is provided, based on what we believe is an important learning effect that the tournament literature has overlooked.

The seminal study of promotion tournaments is the one by Lazear and Rosen (1981). They consider a situation in which two employees compete for a promotion. A key feature of their model is that the employer commits to pay wages (or prizes) to both the promoted and the non-promoted employee before the tournament starts. Lazear and Rosen find that heterogeneity among employees is detrimental from an incentive perspective (unless the employer introduces handicaps to counteract the heterogeneity). This is because heterogeneity lowers the marginal effect of effort on an employee’s probability of being promoted (i.e. of producing higher output than the opponent does). Intuitively, the employee of lower ability realizes that he is unlikely to overcome the ability advantage of the opponent and reduces his effort. Then the employee of higher ability can afford to relax and reduce his effort as well.¹

However, the assumption that the employer can commit to pay different prizes at the beginning of the tournament has come under recent scrutiny. Following Waldman (1984), several tournament papers appeared that restrict the power of the employer to commit to a certain set of prizes.² Instead, it has been argued that post-tournament wages are determined by a bidding process taking into account promotion decisions. In particular, the labor market (i.e. firms other than the current employer) understands promotion as a (positive) signal of an employee’s ability. Accordingly, promotion induces the labor market to upgrade the assessment of an employee’s ability, which consequently leads to higher wage offers for that employee. Therefore, employees have an incentive to vie for promotion.

In such a market-based tournament, employee heterogeneity affects the employees’ payoff function in two ways. On one hand, the effect is the same as that in the model of Lazear and Rosen: heterogeneity lowers the marginal effect of effort on the probability of being

¹See Gürtler and Kräkel (2010).
promoted. On the other hand, heterogeneity has an effect on how the labor market uses promotion decisions to update the assessment of employee ability. In certain situations (i.e. for certain families of ability distributions), ability assessments are more sensitive to promotion decisions when employees are heterogeneous rather than homogeneous. This observation is based on the following intuition: If an employee considered to be of low ability is promoted even though he had to compete against a high-ability employee, his ability assessment is substantially upgraded. Similarly, an employee considered to be of high ability suffers significant downgrading of his ability assessment when he does not compete successfully against a low-ability employee. A significant change in ability assessment leads to a strong change in the wage offered. In a heterogeneous tournament in which ability assessments, and thus wages, are more sensitive to promotion decisions, employees have a higher incentive to win the tournament. This effect may be so strong that it more than compensates the original effect identified by Lazear and Rosen, so that it is optimal for an employer to hire heterogeneous employees.

Some empirical evidence exists that is in line with the findings of the present model. First, there is ample anecdotal evidence of young and relatively unknown athletes "becoming stars overnight" after succeeding against well-known rivals. These include the famous tennis players Boris Becker and Roger Federer, who won the Wimbledon men’s title in 1985 and 2003, respectively, and basketball player Jeremy Lin, who scored 38 points and proved himself instrumental in the victory of the New York Knicks over the Los Angeles Lakers in 2012. These findings indicate that performing well against strong rivals can boost a player’s career and thus substantiate the notion that ability assessments are more sensitive to the realization of relative performance signals for heterogeneous contestants. In addition to this anecdotal evidence, several empirical studies provide support for our results. DeVaro and Waldman (2012) analyze data from a medium-sized US firm in the financial-services industry. They find that, upon promotion, employees with a Masters degree or a Ph.D. receive a smaller wage increase compared to employees with a Bachelors degree. This observation is consistent with the present result that, following a promotion, the ability assessment of low-ability employees is upgraded by a greater degree than that of high-ability employees. Bognanno and Melero (2012), using data from the British Household Panel Survey, obtain similar results: less educated employees receive larger wage increases upon promotion. Observations by DeVaro (2006) provide some indirect support for the model presented here. In an analysis
of promotion decisions using data from the Multi-City Study of Urban Inequality, he finds that employers react to factors that suppress incentives by counteracting and increasing the spread between winner and loser prizes. This finding is in line with our results. As indicated, heterogeneity among employees reduces the marginal effect of effort on the probability of being promoted. This induces employees to reduce their effort. However, as argued before, it is possible that the labor market changes the assessment of employee ability to a greater degree after observing the tournament outcome. As a consequence, the difference in wage offers from external employers to the promoted and non-promoted employee increases. If the current employer matches these external offers, the tendency is to react to higher employee heterogeneity by increasing the spread between prizes. Finally, our model can explain why studies have observed benefits from the design of heterogeneous workgroups (e.g., Hamilton et al. 2003, Franck and Nüesch 2010).

In addition to the literature cited so far, our study is related to the literature on learning in tournaments. This literature focuses on the question of whether tournaments succeed at identifying (and selecting) the most able contestant, in other words, whether the most able contestant wins the tournament. Meyer (1991), for example, considers a series of tournaments between two heterogeneous employees. She demonstrates that selection efficiency can be improved by biasing the tournament results. If only ordinal information about the employees’ performances is available, optimal bias (which in most cases favors the actual leader in the tournament) increases the information content of the tournament such that the information becomes a sufficient statistic for cardinal information. Clark and Riis (2001) show that efficient selection can be achieved by combining a promotion tournament with absolute performance standards. In their model, there are three tournament prizes and the tournament winner receives the highest prize only if his performance surpasses a threshold level. By using performance standards, the employer receives further information about employee ability that he can use to select employees efficiently. Hvide and Kristiansen (2003) emphasize the relevance of the selection problem. They examine a promotion tournament, in which employees can choose strategies that differ in risk. They find that selection efficiency may be low, because low-ability employees might choose risky strategies that overturn their ability disadvantage. Chen (2003) and Münster (2007) allow for sabotage in tournaments. They find that high-ability employees are strongly sabotaged, which in turn leads to low selection
efficiency. Höchtl et al. (2011) consider optimal seeding in an elimination tournament with two low and two high-ability employees. The goal of the firm is to induce employees to exert a high effort and to promote a high-ability employee. The authors demonstrate that these two objectives are conflicting and that seeding employees to induce high efforts leads to low selection efficiency (and vice versa). This result is similar to the main finding of DeVaro and Gürtler (2013), who consider a promotion tournament in which employees perform multiple tasks. Efficient assignment of employees to jobs would require a promotion rule that focuses on the task that is relatively most important in a high-level job. However, such a promotion rule would induce employees to neglect the remaining tasks and thus leads to suboptimal incentives. It should be noted that in all these studies, the employer can either commit to certain tournament prizes at the beginning of the tournament or the prizes are exogenously given. This means that none of the studies assumes that prizes are offered on the basis of a bidding process in the labor market. Obviously, this approach contrasts with that in the current study, in which the employer’s inability to commit to a set of prizes at the beginning of the tournament is a necessary condition for the optimality of employee heterogeneity.

Finally, our analysis is related to Rantakari (2012), who considers a situation in which each employee exerts two-dimensional effort to affect the value of a project for the employer as also for himself. The project that yields a higher value for the employer is realized by the employer. Accordingly, employees engage in a type of competition against each other, because they prefer their own project to be realized. Rantakari demonstrates that the employer may benefit from hiring heterogeneous employees. This finding depends on "winner prizes" being endogenous in the sense that they depend on the employees’ efforts. This is similar to our contention here, whereby the benefit that accrues to employees from winning a heterogeneous tournament is greater than the prize for a homogeneous one. In the present model, however, learning of employee abilities is the driving force behind this result.

The remainder of the paper is organized as follows: Section 2 presents the market-based tournament. Section 3 investigates the matching of workers to firms. Section 4 discusses the findings and Section 5 concludes. All proofs are provided in the Appendix.

3See also Gürtler and Münster (2010) and Gürtler et al. (2013).
2 Market-based tournaments

2.1 Description of the model and notation

We consider a model of a competitive labor market with two periods, \( \tau = 1, 2 \). There are \( N \) identical firms and \( n \) workers, all parties are risk-neutral. Each firm has two different types of job, a low-level job 1 and a high-level job 2. Jobs are indexed by \( k = 1, 2 \). If worker \( j \) is hired by firm \( i \) in period \( \tau \) and assigned to job \( k \), his output is given by

\[
y^k_{ij\tau} = d_k + (1 + s) c_k (e_{j\tau} + t_j + \varepsilon_j) + \gamma_{j\tau}.
\] (1)

Effort is denoted by \( e_{j\tau} \geq 0 \) and ability by \( t_j + \varepsilon_j \) for worker \( j \). It is assumed that all firms and all workers observe \( t_j \) at the beginning of the first period. However, there is symmetric uncertainty for \( \varepsilon_j \) (i.e. none of the parties knows \( \varepsilon_j \) ex ante, as assumed for example in Holmström 1982). We assume that \( \varepsilon_j \) is continuously, identically, and independently distributed. The probability density function (pdf) of \( \varepsilon_j \) is assumed to be symmetric around zero with domain \([-b, b]\). Accordingly, all parties know the expected ability of a worker, \( t_j \) (e.g. by observing the worker’s level of education), but not the actual realization. We impose the condition \( \min \{t_A, t_B\} > b \) so that ability is always positive. \( d_k \) and \( c_k > 0 \) are parameters characterizing worker productivity. Following Waldman (1984), we assume that \( c_2 > c_1 \) (and \( d_2 < d_1 \)), so that output is more responsive to ability in the high-level job. \( \gamma_{j\tau} \) captures random noise affecting the output produced in period \( \tau \). It is continuously, identically, and independently distributed according to a pdf that is assumed to be symmetric around zero with bounded domain. Finally, \( s \in \{0, S\} \) is an indicator variable capturing firm-specific human capital acquired in the first period of employment. Its realization is equal to zero \( (s = 0) \) if the first period is considered or if the second period is considered and worker \( j \) has moved to a different firm after the first period. The variable equals \( S \ (s = S > 0) \) if the second period is considered and the worker continues to work for the same firm as in the first period. Effort is costly to the workers and the effort costs are given by \( c(e_{j1}, e_{j2}) \). Costs are increasing, strictly convex, and satisfy \( \frac{\partial c}{\partial e_{j\tau}} (0, 0) = 0 \) for \( \tau = 1, 2 \).

Throughout Section 2, we restrict our attention to a representative firm that has hired two workers (indexed by \( j = A, B \)) at the beginning of the first period. We assume that the firm decides to assign both workers to the low-level job 1 in \( \tau = 1 \).

\(^4\)Two reasons may account for this type of behavior. First, situations occur in which workers have to work
period, the firm observes the output of each worker and then promotes the worker who has produced the higher output to the high-level job. Other firms cannot observe individual outputs, but can observe which jobs workers have been assigned to at the end of the first period. They use this information to update their ability assessment for the two workers.

At the beginning of the second period, other firms try to hire the two workers by making wage offers. It is assumed that all wage offers (including the one from the current employer) are made simultaneously. Each worker is hired by the firm making the highest offer. Ties are broken randomly except for the case in which the current employer is among the firms offering the highest wage. In this case, the worker remains with the current employer. We assume that the parameter constellations are such that if an external firm were successful in hiring one of the workers, it would always assign this worker to job regardless of whether the worker was assigned to job 1 or job 2 in the initial firm. We further assume that in equilibrium, firms are never successful at hiring workers away from other firms. However, as in Greenwald (1986) and Waldman (2013a), there is a small probability that workers will in the low-level job first to acquire skills that allow them to accomplish the tasks required in the high-level job. Second, we could assume that is rather low, in which case the firm finds it optimal to assign both workers to job 1 in . Implicitly, we assume that the firm always finds it optimal to promote one of the workers (which, for instance is true if is sufficiently high), but that there is a slot constraint in the high-level job, so it is impossible to assign both workers to job 2. It should be noted that the promotion rule is optimal among all conceivable promotion rules. When deciding which worker to promote, the firm takes two different aspects into account. On one hand, it wants to promote the worker with the higher ability (because output is more sensitive to ability in the high-level job); on the other hand, it wants to minimize second-period wage costs. We show that both workers choose the same effort in equilibrium, so that promotion of the worker with higher first-period output is equivalent to promotion of the worker with higher expected ability. Thus, the first objective of the firm is always fulfilled. In addition, it can be shown that total second-period wages do not depend on whether worker A or worker B is promoted (a proof is available from the authors upon request), which demonstrates the optimality of the promotion rule.

This assumption is related to the assumption that the first-period employer assigns the workers to the low-level job in . Since external firms assign workers efficiently in (given their information about abilities) and firm-specific human capital is lost when workers switch employers, assignment of workers to job 1 requires to be rather low. Note, however, that this assumption is not crucial for our results. All the qualitative results still hold if an external firm successful in hiring one of the workers always assigns this worker to the high-level job 2.

Again, this assumption requires to be sufficiently high so that external firms are never able to outbid the current employer.
switch employers after the first period for exogenous reasons that are unrelated to ability and job assignment. As explained in these two papers and in the next subsection, the latter two assumptions eliminate the winner’s-curse effect.

Explicit incentive schemes that link pay to performance are not feasible. Furthermore, long-term contracts that bind workers to the firm for both periods are not feasible either. There is no discounting.

2.2 Model solution

The model is solved by backward induction. Because effort costs increase with the level of effort and the firms cannot use pay-for-performance schemes, all workers who are hired during the second period choose a second-period effort of zero, \( e_{j2} = 0 \).\(^8\) Hence, worker output is either \( y_{ij2}^k = d_k + (1 + S) c_k (t_j + \varepsilon_j) + \gamma_{j2} \) if the worker stays with his current firm or \( y_{ij2}^l = d_1 + c_1 (t_j + \varepsilon_j) + \gamma_{j2} \) if he moves to another firm. The ability assessment for a worker depends on whether he is promoted \( (p_j = 1) \) or not \( (p_j = 0) \), and so does his second-period wage, as indicated by the following lemma (\( E[\cdot|\cdot] \) denotes the conditional expectation operator).\(^9\)

**Lemma 1** In any equilibrium in undominated strategies, the second period wage for worker \( j \) is given by \( d_1 + c_1 E[t_j + \varepsilon_j|p_j = 1] \) if he is promoted at the end of the first period and by \( d_1 + c_1 E[t_j + \varepsilon_j|p_j = 0] \) if he is not.

In the first period, the labor market observes which of the two workers is promoted and uses this information to update the assessment of their abilities. Obviously, the workers vie for promotion, because this increases their ability assessment and their second-period wage.

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\(^8\)The fact that effort is zero is a normalization and should not be taken literally. This should be interpreted as the effort that the workers would choose if there were no (explicit or implicit) incentive pay. Typically, workers exert some "regular" effort level even in the absence of incentive pay, because they experience some utility from working up to a certain point. This regular effort level is normalized to zero in the model. A similar argument is advanced by Holmström and Milgrom (1991) and Grund and Sliwka (2010).

\(^9\)In the wage-setting subgame at the beginning of the second period, there exist equilibria in which workers receive wages that differ from those specified in Lemma 1. As these equilibria involve weakly dominated strategies (and thus do not survive equilibrium refinements such as trembling-hand perfection), we neglect these equilibria in what follows.

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compensation. Consider worker $A$. This worker chooses his first-period effort to maximize

$$U_A = d_1 + c_1 E \left[ t_A + \varepsilon_A \middle| p_A = 1 \right] P (p_A = 1) + c_1 E \left[ t_A + \varepsilon_A \middle| p_A = 0 \right] (1 - P (p_A = 1)) - c(e_{A1})$$

$$= d_1 + c_1 E \left[ t_A + \varepsilon_A \middle| p_A = 0 \right] + c_1 (E \left[ \varepsilon_A \middle| p_A = 1 \right] - E \left[ \varepsilon_A \middle| p_A = 0 \right]) P (p_A = 1) - c(e_{A1}),$$

where $P(\cdot)$ is the probability operator. Note that the labor market cannot observe the efforts of the two workers. Hence, when calculating the expected ability of worker $A$ conditional on second-period job assignment, the labor market has to form a belief regarding these efforts. We denote the efforts that firms believe the two workers choose by $\tilde{e}_{A1}$ and $\tilde{e}_{B1}$ and the difference between them by $\Delta \tilde{e} := \tilde{e}_{B1} - \tilde{e}_{A1}$. In contrast, the promotion probability $P (p_A = 1)$ for worker $A$ depends on the actual effort difference $e := e_{B1} - e_{A1}$. We further define $\Delta t := t_B - t_A$, which serves as our measure of worker heterogeneity, with higher $|\Delta t|$ indicating more heterogeneous workers, and $\gamma := \frac{e_{A1} - e_{B1}}{c_1}$, which is continuously distributed according to a pdf $g$ that is independent of $\varepsilon_j$ and has bounded domain $[-a, a]$. Later, we encounter the random variables $\varepsilon_A - \varepsilon_B + \gamma$ and $\varepsilon_A - \varepsilon_B$. We denote the pdfs of these two random variables by $\phi$ and $f$ and the corresponding cumulative distribution functions (cdfs) by $\Phi$ and $F$, respectively. As is standard in the tournament literature, we assume that $\phi$ and $f$ have a unique mode at zero. Furthermore, we suppose that $f$ is piecewise continuously differentiable on $(-2b, 2b)$, which implies that $\phi$ is continuously differentiable on $(-2b - a, 2b + a)$.\(^\text{10}\) Worker $A$ will be promoted if and only if $e_{A} - e_{B} + \gamma > \Delta e + \Delta t$, so $U_A$ can be rewritten as

$$U_A = d_1 + c_1 (t_A + E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta \tilde{e} + \Delta t])$$

$$+ c_1 (E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta \tilde{e} + \Delta t] - E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta \tilde{e} + \Delta t])$$

$$\cdot (1 - \Phi (\Delta e + \Delta t)) - c(e_{A1}).$$

\(^\text{10}\)Consider the relationship $\Phi(\Delta t) = P(\varepsilon_A - \varepsilon_B + \gamma \leq \Delta t) = E[F(\Delta t - \gamma)]$, where $E[\cdot]$ denotes the expectation operator. This leads to $\phi(\Delta t) = E[f(\Delta t - \gamma)]$ and the piecewise continuous differentiability of $f$ immediately implies the continuous differentiability of $\phi$.\(^\text{9}\)
Optimal effort by worker $A$ is characterized by the following first-order condition for his maximization problem:\textsuperscript{11}

\[
\frac{\partial U_A}{\partial e_{A1}} = c_1 \cdot (E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta \hat{e} + \Delta t] - E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta \hat{e} + \Delta t])
\cdot \phi (\Delta e + \Delta t) - c'(e_{A1}) = 0.
\]

Analogously, for worker $B$ we obtain

\[
\frac{\partial U_B}{\partial e_{B1}} = c_1 \cdot (E [\varepsilon_B | \varepsilon_B - \varepsilon_A - \gamma > -\Delta \hat{e} - \Delta t] - E [\varepsilon_B | \varepsilon_B - \varepsilon_A - \gamma < -\Delta \hat{e} - \Delta t])
\cdot \phi (\Delta e + \Delta t) - c'(e_{B1}) = 0.
\]

In equilibrium, the labor market correctly anticipates the workers' behavior, and thus $\hat{e}_{A1} = e_{A1}$ and $\hat{e}_{B1} = e_{B1}$. Taking all these conditions into account, we obtain the following lemma.

**Lemma 2** In the tournament, a unique equilibrium exists that is symmetric with both workers choosing the same effort $e$. $e$ is implicitly defined by

\[
c'(e) = c_1 \cdot \phi(\Delta t) \cdot W_A(\Delta t),
\]

where $W_A(\Delta t) := E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t] - E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta t]$.

The condition presented in Lemma 2 clearly shows the two different effects that worker heterogeneity ($\Delta t$) has on the incentive to exert effort. On one hand, heterogeneity affects the marginal effect of effort on the probability of winning, $\phi(\Delta t)$. As demonstrated in the literature, because $\phi$ has a unique mode at $\Delta t = 0$, heterogeneity reduces the incentive to exert effort by lowering the marginal effect of effort on the probability of winning. On the other hand, worker heterogeneity affects the ability updating process following the promotion decision. For instance, if a worker who is thought to have low ability performs better than a worker of seemingly higher ability, his ability assessment is upgraded by a higher degree than in a situation in which he performs better than a worker of rather low ability. In turn, when the labor market puts greater emphasis on the promotion decision, workers have a higher

\textsuperscript{11}A typical feature of tournament models is that the cost function must be sufficiently convex for the objective function to be strictly concave and to meet the second-order conditions. In what follows, we assume that this is the case, so that optimal efforts are indeed characterized by the first-order conditions for the maximization problem.
incentive to exert effort to achieve promotion and thus to affect their ability assessment and future compensation. This effect is captured by the term $W_A(\Delta t)$. The following proposition indicates how $W_A(\Delta t)$ changes when workers are heterogeneous rather than homogeneous.

**Proposition 1** Let $\text{Let}^{12}$

$$E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0) \right] \cdot \phi(0) > \frac{1}{8}.$$  

Then $W_A$ has a local minimum at $\Delta t = 0$.

Starting from a situation with homogeneous workers ($\Delta t = 0$), introduction of worker heterogeneity has two countervailing effects on the wage differential for each of the workers considered. Consider the worker with the lower expected ability. If this worker is promoted even though he had to compete against a stronger rival, his ability assessment is significantly upgraded. This effect increases the wage differential and thus the worker’s incentive to exert effort. By the same token, however, downgrading of the worker’s ability assessment if he is reassigned to the low-level job is rather weak, which attenuates his incentive to exert effort.

If the worker with the higher expected ability is promoted, his ability assessment does not increase much, because he had to outperform only a weak rival. Instead, if the worker is not promoted, his ability assessment is strongly downgraded. Again, we observe countervailing effects of heterogeneity on the wage differential and the incentive to exert effort.

Proposition 1 offers conditions under which the positive incentive effects dominate the negative effects, so that $W_A(\Delta t)$ reaches a local minimum at $\Delta t = 0$.\(^{13}\) In particular, both $E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0) \right]$ and $\phi(0)$ are required to be rather high. Note that $E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0) \right]$ is a measure of dispersion of the distribution of $\varepsilon_A - \varepsilon_B$ and $\phi(0)$ is the value of the pdf of $\varepsilon_A - \varepsilon_B + \gamma$ at its zero mean. Suppose we begin at $\Delta t = 0$ and introduce a small amount of heterogeneity by slightly increasing $\Delta t$ to $\Delta t > 0$. Consider the weaker worker. As explained before, promotion leads to a greater change in this worker’s ability assessment, because he has to compete against a stronger rival. More specifically, the ability assessment in a situation in which the worker is promoted increases by $E \left[ \varepsilon_A \mid \varepsilon_A - \varepsilon_B + \gamma > \Delta \hat{t} \right] - E \left[ \varepsilon_A \mid \varepsilon_A - \varepsilon_B + \gamma > 0 \right]$ when heterogeneity is introduced. This difference in ability assessments increases with $\phi(0)$. When $\phi(0)$

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\(^{12}\) $I(A)$ denotes the indicator function, so $I(A) = 1$ if the event $A$ is true, whereas $I(A) = 0$ otherwise.

\(^{13}\) In particular, we show that if the condition from Proposition 1 is met, $W'_A(0) = 0$ and $W''_A(\Delta t) > 0$ for all $\Delta t$ belonging to an open and nonempty interval around zero.
is high, values close to zero (i.e. at the left tail of the conditional distribution) are assigned a significant weight when \( E[\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0] \) is calculated, but are truncated when \( E[\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta \hat{t}] \) is determined. Furthermore, the positive effect of a high value of \( \phi(0) \) on \( E[\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta \hat{t}] - E[\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0] \) is intensified when the dispersion of the random variables is high. This is intuitive. Truncation of the pdf of \( \varepsilon_A - \varepsilon_B + \gamma \) at \( \Delta \hat{t} \) leads to a stronger shift of the considered probability mass to the right tail of the distribution (i.e. to high values of \( \varepsilon_A \)) for greater dispersion of the random variables. As a consequence, the positive incentive effects of an introduction of heterogeneity are particularly strong if both \( E[(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)] \) and \( \phi(0) \) are high.

If the conditions from Proposition 1 are met, a direct implication is that the ability assessment for the workers is more responsive to the promotion decision in the case of heterogeneous workers than in the case of homogeneous workers, at least if heterogeneity is not substantial. As just explained, worker heterogeneity thus has a positive effect on their incentive to exert effort. The following proposition shows that this effect is so strong that it may outweigh the negative effect of heterogeneity on the marginal effect of effort on the probability of winning, inducing workers to choose a higher effort in a heterogeneous tournament.

**Proposition 2** Let the assumptions from Proposition 1 be valid and let \( \phi' \) be continuously differentiable on \((-2b - a, 2b + a)\). Furthermore, let

\[
\frac{W''_A(0)}{W_A(0)} > - \frac{\phi''(0)}{\phi(0)}.
\]

Then optimal effort \( e \) has a local minimum at \( \Delta t = 0 \).

Proposition 2 is very intuitive. It simply states that if the positive effects of worker heterogeneity on the incentive to exert effort dominate the negative ones, effort has a local minimum at \( \Delta t = 0 \). Accordingly, workers exert higher effort in a heterogeneous tournament than in a homogeneous one.

We conclude this section with a brief example that highlights the optimality of heterogeneous tournaments in terms of effort provision. Suppose that \( \varepsilon_A \) and \( \varepsilon_B \) are uniformly distributed on \([-b, b]\), which implies that \( f \) is piecewise differentiable on \([-2b, 2b]\). Suppose further that \( \gamma \) is uniformly distributed on \([-a, a] \) with \( 0 < a < 2b \). It can then be shown that

\[
E[(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)] = \frac{a^3 - 4a^2b + 16b^3}{48b^2},
\]

and

\[
\phi(0) = \frac{4b - a}{8b^2}.
\]
It immediately follows that for a sufficiently close to zero, the condition

\[ E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0) \right] \cdot \phi(0) > \frac{1}{8} \]

is met. Since \( \phi''(0) = 0 \) and \( W''_A(0) > 0 \) the conditions from Propositions 1 and 2 are satisfied, so that equilibrium effort is higher in a situation with heterogeneous workers relative to a situation with homogeneous workers.

### 3 Matching of workers to firms

In this section, we analyze how workers are matched to firms at the beginning of the first period. We assume that each firm can hire exactly two workers or none at all. There are two types of workers, workers with high expected ability \( t_H \) (type 1) and workers with low expected ability \( t_L < t_H \) (type 2). There are \( \frac{n}{2} \) workers of type 1 and \( \frac{n}{2} \) workers of type 2, where \( \frac{n}{2} < N \). We assume that the conditions from Proposition 2 are met and that \( \Delta t^* := t_H - t_L \) is sufficiently small, so that workers choose a higher effort if the other worker hired along with them is of a different type rather than of the same type. Induction of higher effort is beneficial only if effort is not already inefficiently high. Given that the second-period effort is zero, the first-period effort \( e^* \) that maximizes total surplus is characterized by the first-order condition \( e'(e^*) = c_1 \). To ensure that the equilibrium effort is never inefficiently high (so that \( e \leq e^* \)), the following additional assumption is made:

\[ W_A(\Delta t^*) \cdot \phi(\Delta t^*) \leq 1. \quad (A1) \]

The following proposition then demonstrates that the firms decide to hire heterogeneous workers and thus to implement heterogeneous tournaments. This is because heterogeneous tournaments enable the firms to induce the highest efforts (i.e. efforts that are closest to the efficient level).

**Proposition 3** Suppose that the conditions from Proposition 2 hold, let assumption (A1) be fulfilled, and let \( \Delta t^* \) be sufficiently small. Then, in equilibrium, \( \frac{n}{2} \) of the \( N \) firms each hire one worker of type 1 and one worker of type 2, while the remaining firms do not hire a worker.
at all. Workers of type 1 are paid a first-period wage of

\[ w_1 = d_1 + c_1 (t_H + e) \]

\[ + \Phi (\Delta t^*) \cdot (d_2 - d_1 + ((1 + S) c_2 - c_1) \cdot (t_H + E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*])) \]

\[ + (1 - \Phi (\Delta t^*)) \cdot S \cdot c_1 \cdot (t_H + E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*]) \]

and workers of type 2 a wage of

\[ w_2 = d_1 + c_1 (t_L + e) \]

\[ + \Phi (\Delta t^*) \cdot S \cdot c_1 \cdot (t_L + E [\varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*]) \]

\[ + (1 - \Phi (\Delta t^*)) \cdot (d_2 - d_1 + ((1 + S) c_2 - c_1) \cdot (t_L + E [\varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*])) \].

Workers of different types receive different first and second-period wages, so the change in compensation relative to the first period following a win or loss in the tournament may also differ for the workers. Denote the difference between the second-period and first-period wages for a worker of type 1, upon promotion, by \( \Delta_1 = d_1 + c_1 (t_H + E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*]) - w_1 \), and let \( \Delta_2 \) be defined analogously, \( \Delta_2 = d_1 + c_1 (t_L + E [\varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*]) - w_2 \).

We obtain the following proposition.

**Proposition 4** Suppose that the conditions from Proposition 3 hold. Then in equilibrium we observe \( \Delta_1 < \Delta_2 \). This means that the change in compensation upon promotion (relative to the first-period compensation) is higher for a worker of type 2 than for a worker of type 1.

Proposition 2 demonstrates that the incentive to exert effort may be higher in a heterogeneous tournament than in a homogeneous one. Proposition 4 sheds more light on this issue. The proposition shows that workers of type 2 (workers with relatively low ability) have a high incentive to achieve promotion, because winning against a high-ability worker leads to a significant upgrade in ability assessment and thus to a higher wage. On the contrary, the ability assessment of a type 1-worker (worker of higher ability) does not change significantly if he competes successfully against a low-ability worker. Instead, the worker is motivated to exert effort, because his ability assessment would be strongly downgraded if he were to lose against a worker of low ability. Thus, the worker strives hard to avoid losing in the promotion tournament.
4 Discussion

There has been a recent discussion in the literature of the empirical relevance of "classic tournaments" in the spirit of Lazear and Rosen (1981) compared to that of "market-based tournaments" analyzed in the current paper. DeVaro (2011) and Waldman (2013a) discuss possible ways to empirically differentiate between these two types of tournament. The model proposed here opens up another possibility for differentiating between classic and market-based tournaments. Whereas workers in classic tournaments decrease their effort as a response to higher worker heterogeneity, the current paper shows that the reaction of workers in market-based tournaments may be opposite, in that they choose a lower effort in homogeneous tournaments than in heterogeneous tournaments. Thus, by analyzing the reaction of contestants to heterogeneity, the empirical relevance of both types of tournament can be assessed.

Some empirical evidence exists that is in line with our model findings. Using data for a medium-sized US firm in the financial-services industry, DeVaro and Waldman (2012) find that on promotion, employees with a Masters degree or a Ph.D. receive a smaller wage increase compared to the increase for employees with a Bachelors degree. This observation is consistent with Proposition 4 if we interpret education level as a proxy for expected ability. Using data from the British Household Panel Survey, Bognanno and Melero (2012) obtain similar results: less educated employees receive larger wage increases on promotion. However, it should be noted that alternative explanations exist for these findings. DeVaro and Waldman (2012) develop a model without slot constraints for the high-level job, so that only a worker's absolute performance determines the promotion decision (and not the worker's performance relative to that of other workers). As in the current model, promotion serves as a signal regarding a worker's ability, and thus a worker's current employer tends to distort the promotion decision and promotes only workers who have sufficiently high ability. It is found that the promotion decision is less distorted for workers with a high level of education. Moreover, since workers with a high level of education are more productive than workers with a low level of education, the effect of promotion on the labor market assessment of a worker's ability decreases with the level of education of the worker. Both effects imply that wage increases upon promotion are smaller for workers whose level of education is high rather than low. In contrast, in the current model the promotion decision is based on the relative performance of workers. Thus,
the improvement in a worker’s ability assessment upon promotion depends not only on his own expected ability, but also on that of the worker he was competing against. Accordingly, the change in ability assessment is more significant if a worker is promoted even though he had to compete against a rival who is believed to have high ability.

Waldman (2013b) reports on empirical evidence indicating that education is positively related to promotion probability. This finding is consistent with the current model. To understand this, continue to assume that the level of education is a proxy for expected ability. If firms hire homogeneous workers, each worker has a promotion probability of 0.5 regardless of his expected ability and thus his level of education. However, we find that firms hire heterogeneous workers, so that workers with high expected ability are more likely to be promoted than workers with low expected ability. Again, it should be noted that alternative explanations for the positive relation between education and promotion probability exist.  

Some empirical studies analyze the effects of contestant heterogeneity in tournaments. In contrast to the current model, these studies seem to suggest that contestant heterogeneity leads to a lower effort, because of which tournament organizers try to avoid high contestant heterogeneity. Brown (2011) demonstrates that PGA golfers need 0.2 additional strokes on average to complete the first round when Tiger Woods participates in a tournament relative to when he is absent. This can be understood as evidence of lower effort in response to higher player heterogeneity. Knoeber and Thurman (1994) study broiler production, in which producers contract with growers to raise their broiler chickens and reward them depending on their performance relative to other growers. Knoeber and Thurman find evidence that is in line with notions that producers handicap growers of high ability and producers sort growers into homogeneous tournaments. At first sight, these results seem to be inconsistent with the model presented here. However, it should be noted that the present model applies to rather young contestants, about whom little is initially known so that the relative performance signal is used to deduce contestants’ abilities.  

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15 A similar argument is used by Farber and Gibbons (1996) and by Altonji and Pierret (2001), who study employer learning about worker ability. Both studies use data from the National Longitudinal Survey of Youth to test predictions arising from their models, precisely because learning about worker ability is likely to be most important at the beginning of a worker’s career. The finding by DeVaro and Waldman (2012) that employees with a Masters or Ph.D. degree receive a smaller wage increase on promotion compared to that for employees with a Bachelors degree holds for the first promotion, but not for subsequent promotions.
contestants, regardless of their age. An empirical test of the model requires that the field of contestants be segregated by age. For young contestants, learning of their ability is important so that contestant heterogeneity increases the incentive to exert effort, as shown in the present model. In contrast, the characteristics of older contestants may already be well known. Here, heterogeneity is expected to have a negative impact on effort, as demonstrated in "classic tournament models". Another important model assumption is that external employers can observe the field of contestants to assess whether a worker had to compete against strong or weak rivals. Accordingly, the model is most likely to refer to firms or industries that are rather transparent. In many cases, firms provide information on their websites about people working in different divisions. Even if this type of information is rather crude (sometimes only the employee names are given), it could be complemented by information gathered from other sources such as LinkedIn. In this way, a fairly accurate picture of the set of contestants could be constructed.

5 Conclusion

In this paper, we investigate market-based tournaments in which firms use promotion decisions to estimate workers' abilities. Workers have an incentive to exert effort to achieve promotion, because promotion has a positive impact on their ability assessment. We demonstrate that ability assessments are more sensitive to promotion decisions when workers are heterogeneous rather than homogeneous. Therefore, in a heterogeneous tournament, workers may exert higher effort, because they have a stronger incentive to affect the tournament outcome. Hiring of heterogeneous workers is then optimal for firms.

More generally, the latter finding implies that policies aimed at "leveling the playing field" are not always as beneficial as they may appear. If workers succeed in spite of many

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16In addition, at least some of the rewards are fixed before the start of a tournament in professional golf. While a golfer's future compensation (e.g. through endorsement deals) may depend on how firms assess the golfer's ability, the prizes that can be won in a given tournament are fixed and do not depend on ability assessments. This latter effect may of course confound the results and lead to a negative relation between contestant heterogeneity and the incentive to exert effort. Formally, if the tournament winner receives a prize of $w$ in addition to the prize we have determined in our model, then the inequality in Proposition 2 becomes

$$\frac{W^a(0)}{W^a(0) + w} > -\frac{\phi''(0)}{2z(0)},$$

which is more difficult to satisfy than the original condition.
obstacles, the labor market learns a lot about their characteristics, so it can reward the workers generously on this basis. This may induce workers to exert much greater effort than when the playing field is a leveled one.
Appendix

Proof of Lemma 1. First, it is to be noted that the expected ability of workers that are actually switching firms is equal to the overall expected ability of workers (conditional on their job assignment at the end of period 1, i.e. $E[t_j + \varepsilon_j | p_j = 1]$ or $E[t_j + \varepsilon_j | p_j = 0]$). This is because of the assumption that in equilibrium workers are never successfully hired away, but a small fraction of workers leaves the first-period employer for reasons that are unrelated to ability and job assignment.

Consider worker $j$ and suppose that this worker has been promoted at the end of the first period ($p_j = 1$). Denote by $w_2$ the highest wage offer to the worker in $\tau = 2$. Suppose that $w_2 < d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1]$. If worker $j$’s first-period employer is among the highest bidders, he retains the worker for sure. Then, one of the other firms would gain by deviating and offering a wage from the interval $(w_2, d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1])$. If worker $j$’s first-period employer is not among the highest bidders, the first-period employer gains from deviating and offering a wage from the interval $[w_2, d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1])$. Thus, in equilibrium we never observe $w_2 < d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1]$.

Note that, for any of the external firms, the offer of a wage above $d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1]$ is (weakly) dominated by the offer of a wage equal to $d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1]$. This means that in any equilibrium in undominated strategies none of the external firms offers a wage above $d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1]$. Then, of course, worker $j$’s current employer does not find it optimal to pay a wage above $d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1]$ either so that $w_2$ never exceeds $d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1]$.

Finally, it is very easy to confirm the existence of an equilibrium in which the worker receives $w_2 = d_1 + c_1 E[t_j + \varepsilon_j | p_j = 1]$. For instance, a situation in which all the firms offer such a wage represents an equilibrium.

The analysis is completely analogous for a worker who has not been promoted at the end of the first period. ■

Proof of Lemma 2. Because $\varepsilon_A$ and $\varepsilon_B$ are i.i.d., the symmetry of $\varepsilon_A$ and $\varepsilon_B$ implies $\varepsilon_A$
and $-\varepsilon_B$ as well as $\varepsilon_B$ and $-\varepsilon_A$ are i.i.d. This in turn leads to

$$E [\varepsilon_B | \varepsilon_B - \varepsilon_A - \gamma > -\Delta \tilde{e} + \Delta t] - E [\varepsilon_B | \varepsilon_B - \varepsilon_A - \gamma < -\Delta \tilde{e} + \Delta t]$$

$$= E [-\varepsilon_A | -\varepsilon_A + \varepsilon_B - \gamma > -\Delta \tilde{e} - \Delta t] - E [-\varepsilon_A | -\varepsilon_A + \varepsilon_B - \gamma < -\Delta \tilde{e} - \Delta t]$$

$$= E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta \tilde{e} + \Delta t] - E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta \tilde{e} + \Delta t].$$

Consequently, the first-order conditions to the workers' maximization problems immediately imply $c'(e_A) = c'(e_B)$. By the strict convexity of $c$ it then follows that $e_{A1} = e_{B1} = e$, which means that the equilibrium is unique and symmetric. The rest of the lemma follows from inserting $e_{A1} = e_{B1}$ and $\Delta \tilde{e} = 0$ into one of the first-order conditions. $\blacksquare$

**Proof of Proposition 1.** Because $\varepsilon_A$ and $\varepsilon_B$ are i.i.d., we have

$$0 = E [\varepsilon_A - \varepsilon_B]$$

$$= E [\varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > \Delta t] (1 - P(\varepsilon_A - \varepsilon_B + \gamma \leq \Delta t))$$

$$+ E [\varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < \Delta t] P(\varepsilon_A - \varepsilon_B + \gamma \leq \Delta t)$$

$$= E [\varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > \Delta t]$$

$$- \Phi(\Delta t) (E [\varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > \Delta t] - E [\varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < \Delta t]).$$

Because $\varepsilon_A$ and $\varepsilon_B$ are further symmetrically distributed and $\gamma$ is independent of $\varepsilon_A$ and $\varepsilon_B$ we obtain

$$E \left[ \varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > \Delta t \right]$$

$$= E \left[ \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t \right] + E \left[ -\varepsilon_B | (-\varepsilon_B) - (-\varepsilon_A) + \gamma > \Delta t \right]$$

$$= 2E \left[ \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t \right]$$

which in turn implies

$$W_A(\Delta t) = E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t] - E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta t]$$

$$= \frac{1}{2} (E [\varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > \Delta t] - E [\varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < \Delta t])$$

$$= \frac{1}{2} E [\varepsilon_A - \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > \Delta t]$$

$$= \frac{1}{2} \frac{\Phi(\Delta t)}{\Phi(\Delta t)(1 - \Phi(\Delta t))}.$$  (3)
On one hand, we have
\[
\frac{\partial [\Phi(\Delta t)(1 - \Phi(\Delta t))]}{\partial \Delta t} = \phi(\Delta t)(1 - 2\Phi(\Delta t)).
\] (4)

On the other hand, it follows
\[
E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > \Delta t) \right]
= \int \int_{-a}^{2b} \varepsilon \cdot I(\varepsilon > \Delta t - \gamma) \cdot f(\varepsilon) d\varepsilon \cdot g(\gamma) d\gamma
= \int \int_{-a}^{2b} \varepsilon \cdot f(\varepsilon) d\varepsilon \cdot g(\gamma) d\gamma
\]
and consequently
\[
\frac{\partial E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > \Delta t) \right]}{\partial \Delta t} = -E \left[ (\Delta t - \gamma) \cdot f(\Delta t - \gamma) \right].
\] (5)

Under consideration of (3), (4), and (5), the derivative of \( W_A \) with respect to \( \Delta t \) can be stated as
\[
W'_A(\Delta t) = \frac{-E \left[ (\Delta t - \gamma) \cdot f(\Delta t - \gamma) \right]}{2 \cdot [\Phi(\Delta t) \cdot (1 - \Phi(\Delta t))]^2} \cdot \Phi(\Delta t) \cdot (1 - \Phi(\Delta t))
+ \frac{-E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > \Delta t) \right]}{2 \cdot [\Phi(\Delta t) \cdot (1 - \Phi(\Delta t))]^2} \cdot \phi(\Delta t) \cdot (1 - 2 \cdot \Phi(\Delta t)) = \frac{\alpha(\Delta t)}{\beta(\Delta t)}.
\]
Because \( \Phi(0) = \frac{1}{2} \) and \( E \left[ \gamma \cdot f(\gamma) \right] = 0 \), it immediately follows \( W'_A(\Delta t = 0) = 0 \). Consequently, it is sufficient to show that \( W''_A(\Delta t) > 0 \) in a neighborhood of \( \Delta t = 0 \). We get
\[
\alpha'(\Delta t) = -E \left[ f(\Delta t - \gamma) \right] \cdot \Phi(\Delta t) \cdot (1 - \Phi(\Delta t))
- E \left[ (\Delta t - \gamma) \cdot f'(\Delta t - \gamma) \right] \cdot \Phi(\Delta t) \cdot (1 - \Phi(\Delta t))
- E \left[ (\Delta t - \gamma) \cdot f(\Delta t - \gamma) \right] \cdot \phi(\Delta t) \cdot (1 - 2 \cdot \Phi(\Delta t))
+ E \left[ (\Delta t - \gamma) \cdot f(\Delta t - \gamma) \right] \cdot \phi(\Delta t) \cdot (1 - 2 \cdot \Phi(\Delta t))
- E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > \Delta t) \right] \cdot \phi'(\Delta t) \cdot (1 - 2 \cdot \Phi(\Delta t))
+ 2 \cdot E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > \Delta t) \right] \cdot \phi^2(\Delta t)
= -[E \left[ f(\Delta t - \gamma) \right] + E \left[ (\Delta t - \gamma) \cdot f'(\Delta t - \gamma) \right] \cdot \Phi(\Delta t) \cdot (1 - \Phi(\Delta t))
- E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > \Delta t) \right] \cdot \phi'(\Delta t) \cdot (1 - 2 \cdot \Phi(\Delta t))
+ 2 \cdot E \left[ (\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > \Delta t) \right] \cdot \phi^2(\Delta t)].
\] (6)
At $\Delta t = 0$ and under consideration of $\Phi(0) = \frac{1}{2}$ and the symmetry properties of $f$, we obtain
\[
\alpha'(0) > 0
\]
\[
\iff - \frac{1}{4} \cdot [E[f(\gamma)] + E[\gamma \cdot f'(\gamma)] + 2 \cdot E[(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)] \cdot \phi^2(0) > 0.
\]
Because $f$ has a unique mode at zero, it follows that $E[f(\gamma)] < 0$, so that the preceding condition is always met if
\[
E[(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)] \cdot \phi(0) > \frac{1}{8}.
\]
(7)
The latter inequality results from the identity $\phi(0) = E[f(\gamma)]$. Consequently, if condition (7) is fulfilled, we have established the existence of $\varepsilon > 0$ such that $\alpha'(<\Delta t) > 0$ and consequently $sgn(\alpha(<\Delta t)) = sgn(\Delta t)$ for all $\Delta t \in (-\varepsilon, \varepsilon)$. Because
\[
W''_A(<\Delta t) = \frac{\alpha'(<\Delta t) \cdot \beta(<\Delta t) - \alpha(<\Delta t) \cdot \beta'(<\Delta t)}{\beta^2(<\Delta t)}
\]
and $\beta(<\Delta t) > 0$, it remains to show that $sgn(\beta(<\Delta t)) = -sgn(\Delta t)$, which immediately follows from $\beta'(<\Delta t) = 4 \cdot [\Phi(<\Delta t) \cdot (1 - \Phi(<\Delta t))] \cdot \phi(<\Delta t) \cdot (1 - 2 \cdot \Phi(<\Delta t))$. \vspace{0.1cm}

\textbf{Proof of Proposition 2.} Due to the implicit definition of $e$ according to $c'(e)/c_1 = W_A(<\Delta t) \cdot \phi(<\Delta t)$ and the convexity of the cost function $c$ it is sufficient to show that the right-hand side of the latter equation has a local minimum at $<\Delta t = 0$. The first derivative of the right-hand side with respect to $<\Delta t$ corresponds to
\[
W'_A(<\Delta t) \cdot \phi(<\Delta t) + W_A(<\Delta t) \cdot \phi'(<\Delta t)
\]
and the second derivative is
\[
W''_A(<\Delta t) \cdot \phi(<\Delta t) + W'_A(<\Delta t) \cdot \phi'(<\Delta t)
\]
\[
+ W_A(<\Delta t) \cdot \phi'(<\Delta t) + W_A(<\Delta t) \cdot \phi''(<\Delta t)
\]
\[
\xrightarrow{\Delta t \to 0} W''_A(0) \cdot \phi(0) + W_A(0) \cdot \phi''(0).
\]
The latter convergence results from the symmetry of $\phi$ which implies $\phi'(0) = 0$. Consequently, because the term (8) is zero if $<\Delta t = 0$ and the latter term is positive by the assumption of the proposition, the function $W_A(<\Delta t) \cdot \phi(<\Delta t)$ has a local minimum at $<\Delta t = 0$. \vspace{0.1cm}

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Proof of Proposition 3. Because the labor market is competitive, we will never observe that a worker is not hired at all. The proof is by way of contradiction. Suppose, therefore, that one of the firms has hired two workers of high ability. Then there must be another firm that has hired two workers of low ability. Denote the expected total payoff of firm and the two workers in these two situations (i.e. the expected total surplus) by $T_{SHH}$ and $T_{SLL}$, respectively. Denote the corresponding surplus in a situation, in which a firm has hired two workers of different type by $T_{SHL}$. If $T_{SHL} > 0.5 (T_{SHH} + T_{SLL})$, two firms that have not hired a worker so far, could each hire one worker away from the two other firms (the firms could always offer a more attractive wage contract than the two other firms could offer in return), contradicting the assumption that a firm manages to hire two workers of the same type in equilibrium.\(^1\) $T_{SHL} > 0.5(T_{SHH} + T_{SLL})$ can be stated as

\[
d_1 + c_1 (t_H + e(\Delta t^*)) - c (e(\Delta t^*)) \\
+ (1 - \Phi (-\Delta t^*)) (d_2 + (1 + S) c_2 E [t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*]) \\
+ \Phi (-\Delta t^*) (d_1 + (1 + S) c_1 E [t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*]) \\
+ (1 - \Phi (-\Delta t^*)) (d_1 + (1 + S) c_1 E [t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*]) \\
+ \Phi (-\Delta t^*) (d_2 + (1 + S) c_2 E [t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*]) \\
> d_1 + c_1 (t_H + e(0)) - c (e(0)) \\
+ 0.5 (d_2 + (1 + S) c_2 E [t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0]) \\
+ 0.5 (d_1 + (1 + S) c_1 E [t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < 0]) \\
+ d_1 + c_1 (t_L + e(0)) - c (e(0)) \\
+ 0.5 (d_2 + (1 + S) c_2 E [t_L + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0]) \\
+ 0.5 (d_1 + (1 + S) c_1 E [t_L + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < 0]).
\]

\(^1\)Even if $T_{SHL} > 0.5 (T_{SHH} + T_{SLL})$, it is not fully clear that a single firm could hire one worker away from each of the two other firms. We therefore focus on two firms and show that these two firms manage to hire workers away from the other two firms, given that $T_{SHL} > 0.5 (T_{SHH} + T_{SLL})$. Doing this, we demonstrate that there is no coalition-proof equilibrium, in which one firm hires two workers of high expected ability (and another firm hires two workers of low expected ability), i.e. we use coalition-proofness as an equilibrium refinement.
Under consideration of $1 - \Phi(-\Delta t^*) = \Phi(\Delta t^*)$ this condition can be restated as

$$2c_1 \left( e(\Delta t^*) - e(0) \right) - 2 \left( c(e(\Delta t^*)) - c(e(0)) \right)$$

$$+ \Phi(\Delta t^*) (1 + S) c_2 E \left[ t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right]$$

$$+ (1 - \Phi(\Delta t^*)) (1 + S) c_1 E \left[ t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right]$$

$$+ \Phi(\Delta t^*) (1 + S) c_1 E \left[ t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right]$$

$$+ (1 - \Phi(\Delta t^*)) (1 + S) c_2 E \left[ t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right]$$

$$> 0.5 (1 + S) c_2 E \left[ t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0 \right] + 0.5 (1 + S) c_1 E \left[ t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < 0 \right]$$

$$+ 0.5 (1 + S) c_2 E \left[ t_L + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0 \right] + 0.5 (1 + S) c_1 E \left[ t_L + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < 0 \right].$$

Notice that $2c_1 \left( e(\Delta t^*) - e(0) \right) - 2 \left( c(e(\Delta t^*)) - c(e(0)) \right)$ is strictly positive. Thus, the preceding condition is met if

$$\Phi(\Delta t^*) c_2 E \left[ t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right]$$

$$+ (1 - \Phi(\Delta t^*)) c_1 E \left[ t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right]$$

$$+ \Phi(\Delta t^*) c_1 E \left[ t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right]$$

$$+ (1 - \Phi(\Delta t^*)) c_2 E \left[ t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right]$$

$$> 0.5 c_2 E \left[ t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0 \right]$$

$$+ 0.5 c_1 E \left[ t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < 0 \right]$$

$$+ 0.5 c_2 E \left[ t_L + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0 \right]$$

$$+ 0.5 c_1 E \left[ t_L + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < 0 \right],$$

which is equivalent to

$$t_H \cdot \left( \Phi(\Delta t^*) c_2 + (1 - \Phi(\Delta t^*)) c_1 - 0.5 (c_1 + c_2) \right)$$

$$+ t_L \cdot \left( (1 - \Phi(\Delta t^*)) c_2 + \Phi(\Delta t^*) c_1 - 0.5 (c_1 + c_2) \right)$$

$$+ \Phi(\Delta t^*) c_2 E \left[ \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right]$$

$$+ (1 - \Phi(\Delta t^*)) c_1 E \left[ \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right]$$

$$+ \Phi(\Delta t^*) c_1 E \left[ \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right]$$

$$+ (1 - \Phi(\Delta t^*)) c_2 E \left[ \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right]$$

$$- c_2 E \left[ \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0 \right] - c_1 E \left[ \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < 0 \right] > 0.$$
Using $E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*] = -E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < - \Delta t^*]$, $E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*] = -E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t^*]$, $E [\varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*] = E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta t^*]$ and $E [\varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*] = E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t^*]$, the preceding inequality becomes

$$
\Delta t^* \cdot (\Phi (\Delta t^*) - 0.5) (c_2 - c_1) - \Phi (\Delta t^*) (c_2 - c_1) E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta t^*] + (1 - \Phi (\Delta t^*)) (c_2 - c_1) E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t^*]
$$

$$
- c_2 E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0] - c_1 E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < 0] > 0.
$$

This condition is equivalent to

$$
\Delta t^* \cdot (\Phi (\Delta t^*) - 0.5) + \Phi (\Delta t^*) E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta t^*] + (1 - \Phi (\Delta t^*)) E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t^*]
$$

$$
- 2 \Phi (\Delta t^*) E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta t^*] - E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0] > 0
$$

$$
\Leftrightarrow \Delta t^* \cdot (\Phi (\Delta t^*) - 0.5) + 2 \Phi (\Delta t^*) E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*]
$$

$$
- E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > 0] > 0.
$$

In addition, note that

$$
0 = E [\varepsilon_A]
$$

$$
= E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*] (1 - \Phi (\Delta t^*)) + E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < - \Delta t^*] \Phi (\Delta t^*)
$$

$$
= E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*]
$$

$$
- \Phi (\Delta t^*) (E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*] - E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < - \Delta t^*])
$$

$$
= E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*]
$$

$$
- \Phi (\Delta t^*) (-E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < \Delta t^*] + E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t^*])
$$

$$
= E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*] - \Phi (\Delta t^*) W_A (\Delta t^*),
$$

which implies

$$
E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > - \Delta t^*] = \Phi (\Delta t^*) W_A (\Delta t^*).
$$

Inserting this expression into the preceding inequality, we obtain

$$
Z (\Delta t^*) := \Delta t^* (\Phi (\Delta t^*) - 0.5) + 2 \Phi (\Delta t^*) \Phi (\Delta t^*) W_A (\Delta t^*) - 0.5 W_A (0) > 0.
$$

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Note that \( Z(0) = 0 \),

\[
Z' (\Delta t^*) = \Phi(\Delta t^*) - 0.5 + \Delta t^* \phi(\Delta t^*) + 2(\phi(\Delta t^*) + \Phi(\Delta t^*) + \Phi(\Delta t^*) \phi(-\Delta t^*)) W_A(\Delta t^*) \\
+ 2\Phi(\Delta t^*) \Phi(-\Delta t^*) W_A'(\Delta t^*)
\]

\[
Z'' (\Delta t^*) = \Phi(\Delta t^*) - 0.5 + \Delta t^* \phi(\Delta t^*) + 2(\phi(\Delta t^*) + (1 - 2\Phi(\Delta t^*))) W_A(\Delta t^*) \\
+ 2\Phi(\Delta t^*) \Phi(-\Delta t^*) W_A'(\Delta t^*)
\]

and

\[
Z'''' (\Delta t^*) = 2\phi(\Delta t^*) + \Delta t^* \phi'(\Delta t^*) + 2(\phi'(\Delta t^*) + (1 - 2\Phi(\Delta t^*)) - 2\phi^2(\Delta t^*)) W_A(\Delta t^*) \\
+ 2\phi(\Delta t^*) (1 - 2\Phi(\Delta t^*)) W_A'(\Delta t^*) + 2(\phi(\Delta t^*) + \Phi(\Delta t^*) - \Phi(\Delta t^*) \phi(-\Delta t^*)) W_A'(\Delta t^*) \\
+ 2\Phi(\Delta t^*) \Phi(-\Delta t^*) W_A''(\Delta t^*)
\]

It follows that \( Z'(0) = 0 \) and \( Z''(0) = 2\phi(0) - 4\phi^2(0) W_A(0) + 0.5 W''^2(0) \). In the proof of Proposition 1 we have demonstrated that

\[
0.5 W''^2(0) > 0.5 \dfrac{\alpha'(0)}{\beta(0)}
\]

\[
= 4 \left( -\frac{1}{4} \cdot [\phi(0) + E [\gamma \cdot f'(\gamma)] + 2 \cdot E [(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)] \cdot \phi^2(0) \right)
\]

\[
> -\phi(0) + 8 \cdot E [(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)] \cdot \phi^2(0).
\]

In addition, we have shown that

\[
W_A(0) = 2E [(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)].
\]

Using these two conditions, we can conclude that

\[
Z''(0) > 2\phi(0) - 8 \cdot \phi^2(0) \cdot E [(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)] \\
-\phi(0) + 8 \cdot \phi^2(0) \cdot E [(\varepsilon_A - \varepsilon_B) \cdot I(\varepsilon_A - \varepsilon_B + \gamma > 0)]
\]

\[
= \phi(0) > 0.
\]

To sum up, for sufficiently small \( \Delta t^* \) (as assumed in this section), the preceding inequality is always satisfied.
We therefore have established that \( \frac{n}{N} \) of the \( N \) firms each hire one worker of type 1 and one worker of type 2. Since the labor market is competitive, total expected profit of the firms over both periods is zero. For a worker of type 1, we therefore observe that

\[
d_1 + c_1(t_H + e) - w_1 + \Phi(\Delta^*)(d_2 - d_1 + (c_2(1 + S) - c_1)E[t_H + \varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > -\Delta^*])
\]

and

\[
(1 - \Phi(\Delta^*))c_1SE[t_H + \varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma < -\Delta^*] = 0,
\]

from which we obtain \( w_1 \) as specified in Proposition 3. \( w_2 \) is determined analogously. \( \blacksquare \)

**Proof of Proposition 4.** Inserting \( w_1 \) and \( w_2 \) as specified in Proposition 3 into

\[
\Delta_1 = d_1 + c_1E[t_H + \varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > -\Delta^*] - w_1
\]

and

\[
\Delta_2 = d_1 + c_1E[t_L + \varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > \Delta^*] - w_2,
\]

we obtain

\[
\Delta_1 = c_1(E[\varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > -\Delta^*] - e)
\]

\[
- \Phi(\Delta^*)(d_2 - d_1 + (c_2(1 + S) - c_1)E[t_H + \varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > -\Delta^*])
\]

and

\[
\Delta_2 = c_1(E[\varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > \Delta^*] - e)
\]

\[
- \Phi(\Delta^*)Sc_1E[t_L + \varepsilon_B|\varepsilon_A - \varepsilon_B + \gamma > \Delta^*]
\]

\[
- (1 - \Phi(\Delta^*))(d_2 - d_1 + ((1 + S)c_2 - c_1)E[t_L + \varepsilon_B|\varepsilon_A - \varepsilon_B + \gamma < \Delta^*]).
\]

It follows that

\[
\Delta_1 < \Delta_2 \iff \Delta_2 - \Delta_1 > 0
\]

\[
\iff c_1(E[\varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > \Delta^*] - E[\varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > -\Delta^*])
\]

\[
+ c_1S \left( (1 - \Phi(\Delta^*))E[t_H + \varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma < -\Delta^*] - \Phi(\Delta^*)E[t_L + \varepsilon_B|\varepsilon_A - \varepsilon_B + \gamma > \Delta^*] \right)
\]

\[
- (2\Phi(\Delta^*) - 1)(d_1 - d_2)
\]

\[
+ (c_2(1 + S) - c_1) \left( \Phi(\Delta^*)E[t_H + \varepsilon_A|\varepsilon_A - \varepsilon_B + \gamma > -\Delta^*] - (1 - \Phi(\Delta^*))E[t_L + \varepsilon_B|\varepsilon_A - \varepsilon_B + \gamma < \Delta^*] \right) > 0.
\]
The inequality can be rewritten as

\[
\begin{align*}
&c_1 (E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > \Delta t^*] - E [\varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*]) \\
+ &S \left( c_1 \cdot \left( (1 - \Phi (\Delta t^*)) E [t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*] - \Phi (\Delta t^*) E [t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*] \right) \\
+ &c_2 \cdot \left( \Phi (\Delta t^*) E [t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*] - (1 - \Phi (\Delta t^*)) E [t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*] \right) \\
+ & (c_2 - c_1) \left( \Phi (\Delta t^*) E [t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*] - (1 - \Phi (\Delta t^*)) E [t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*] \right) > 0.
\end{align*}
\]

Obviously, the first term is strictly positive. It remains to be shown that the sum of the second, third and fourth term is non-negative. We have assumed that the firm always decides to promote one of the workers to the high-level job at the end of the first period. A necessary (but not sufficient) condition for the optimality of this kind of behavior is that (see footnote 5)

\[
d_1 + (1 + S) c_1 (t_L - b) \leq d_2 + (1 + S) c_2 (t_L - b) \Longleftrightarrow d_1 - d_2 \leq (1 + S) (t_L - b) (c_2 - c_1),
\]

which implies

\[
(2 \Phi (\Delta t^*) - 1) (d_1 - d_2) \leq (2 \Phi (\Delta t^*) - 1) (t_L - b) (c_2 - c_1) + (2 \Phi (\Delta t^*) - 1) S (t_L - b) (c_2 - c_1).
\]

Note that

\[
(c_2 - c_1) \left( \Phi (\Delta t^*) E [t_H + \varepsilon_A | \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^*] - (1 - \Phi (\Delta t^*)) E [t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*] \right) > (c_2 - c_1) (2 \Phi (\Delta t^*) - 1) E [t_L + \varepsilon_B | \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^*] > (c_2 - c_1) (2 \Phi (\Delta t^*) - 1) (t_L - b).
\]
It remains to be shown that
\[
S \left( \begin{array}{c}
    c_1 \cdot \left( 1 - \Phi (\Delta t^*) \right) E \left[ t_H + \varepsilon_A \mid \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right] \\
    - \Phi (\Delta t^*) E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right] \\
    + c_2 \cdot \left( 1 - \Phi (\Delta t^*) \right) E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right]
\end{array} \right) \\
\geq (2\Phi (\Delta t^*) - 1) S (t_L - b) (c_2 - c_1).
\]

Note that the expression on the left-hand-side of the inequality is strictly bigger than
\[
S \left( \begin{array}{c}
    c_1 \cdot \left( 1 - 2\Phi (\Delta t^*) \right) E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right] \\
    - \Phi (\Delta t^*) E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right] \\
    + c_2 \cdot (2\Phi (\Delta t^*) - 1) E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right]
\end{array} \right) \\
= S \left( 2\Phi (\Delta t^*) - 1 \right) \left( -c_1 E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right] \\
+ c_2 E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right] \right) \\
> S \left( 2\Phi (\Delta t^*) - 1 \right) \left( -c_1 E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right] \\
+ c_2 E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma > -\Delta t^* \right] \right) \\
= S \left( 2\Phi (\Delta t^*) - 1 \right) (c_2 - c_1) E \left[ t_L + \varepsilon_B \mid \varepsilon_A - \varepsilon_B + \gamma < -\Delta t^* \right] \\
> S \left( 2\Phi (\Delta t^*) - 1 \right) (c_2 - c_1) (t_L - b).
\]

This completes the proof of the proposition. ■
References


