Journal of Banking & Finance 35 (2011) 1683–1697

Contents lists available at ScienceDirect

Journal of Banking & Finance

journal homepage: www.elsevier.com/locate/jbf

Portfolio insurance and prospect theory investors: Popularity and optimal design of capital protected financial products

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ARTICLE INFO

Article history:
Received 17 April 2010
Accepted 14 November 2010
Available online 2 December 2010

JEL classification:
G11
G13

Keywords:
Portfolio insurance strategies
Prospect theory
Monte Carlo simulation
Historical simulation
Financial product design

ABSTRACT

Portfolio insurance strategies are used on both the institutional and the retail side of the asset management industry. While standard utility theory struggles to provide an explanation, this study justifies the popularity of portfolio insurance strategies in a behavioral finance context. We run Monte Carlo simulations as well as historical simulations for popular portfolio insurance strategies and benchmark strategies in order to evaluate the outcomes using cumulative prospect theory. Our simulation results indicate that most portfolio insurance strategies are the preferred investment strategy for a prospect theory investor. Moreover, the analysis provides insights into how portfolio insurance products should be designed and structured to meet the preferences of prospect theory investors as accurately as possible.

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1. Introduction

Portfolio insurance strategies gained momentum with the introduction of synthetic put strategies in the early 1980s by Rubinstein and Leland (1981). Strategies that provide protection against losses, while preserving some upward potential, seem to be attractive for a wide range of investors. On the one hand, institutional investors often use portfolio insurance strategies in tailor-made solutions to protect the stock market exposure of their portfolios against large losses. On the other hand, there are many retail products that guarantee private investors downside protection. In fact, the turbulent stock market behavior in the recent past has demonstrated that portfolio insurance strategies help to avoid significant losses. However, the opportunity costs in terms of limited participation from positive stock market returns are high in normal and good states. Given that taking over systematic risk is rewarded with an equity risk premium, the question arises whether portfolio insurance strategies can ever make sense (Dreher, 1988).

Several theoretical studies examine the optimality of portfolio insurance strategies. For example, Dybvig (1988) analyzes the efficiency of a simple stop-loss strategy with the payoff distribution pricing model. He concludes that the costs of such a strategy can be substantial and should not be ignored by practitioners. This result is due to the fact that a stop-loss strategy (like other dynamic portfolio strategies) is incompletely diversified over time. Benninga and Blume (1985) document that the optimality of a portfolio insurance strategy depends on the investor's utility function. They argue that portfolio insurance with put options is only utility maximizing in incomplete markets (e.g., when an investor is prohibited from investing in the risk-free asset). Black and Perold (1992) prove that the constant proportion portfolio insurance (CPPI) strategy with unconstrained borrowing is utility maximizing only for a HARA utility function. However, borrowing constraints will be imposed in the asset management practice. In this case, the utility maximizing property holds under additional and very restrictive assumptions. Overall, it seems that the unbroken popularity of portfolio strategies cannot be justified based on standard utility theory.

In another strand of the literature, simulation methods are used to analyze the properties of portfolio insurance strategies. Comparing different protection strategies, Benninga (1990) documents that the simple stop-loss strategy tends to dominate more sophisticated portfolio insurance strategies in terms of both the expected terminal wealth and the Sharpe-ratio. Bird et al. (1990) report that standard portfolio insurance strategies are robust to a variety of market conditions, including stock market crashes. More recently, Do (2002) uses simulation analysis to compare the synthetic put

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doi:10.1016/j.jbankfin.2010.11.012

strategy with the CPPI strategy. Although he claims that neither strategy can be justified based on either a loss minimization or a gain participation point of view, the CPPI strategy seems to dominate in terms of floor protection and the costs of insurance. The simulation results in Cesari and Cremonini (2003) indicate that the relative performance of portfolio insurance strategies depends on the market phase. For example, using a variety of performance measures (e.g., the downside deviation and the Sortino ratio), they report a dominant role of the CPPI strategy against all other portfolio insurance strategies in bear and sideway markets. Finally, Annaert et al. (2009) use the concept of stochastic dominance to compare portfolio insurance strategies with buy-and-hold strategies. They cannot identify a dominance relationship between portfolio insurance strategies and buy-and-hold benchmarks. Although portfolio insurance strategies yield lower returns than the benchmark strategies, Annaert et al. (2009) conclude that their accompanying lower risk compensates sufficiently to make them attractive alternatives at least for some investors.

Presumably, the positive framework of behavioral finance offers an explanation for the widespread popularity of portfolio insurance strategies. Experimental evidence indicates that investors are more sensitive to losses than to gains (loss aversion). Moreover, individuals have a tendency to overweight extreme, but unlikely events (e.g., very large, but rare losses). Accordingly, Shefrin and Statman (1993) and Kahneman and Riepe (1998) argue that investment advisors should consider strategies that limit the downside potential while retaining some of the possible upside gains. In fact, generating a kinked return profile in a systematic and rules-based way is the main property of any portfolio insurance strategy.

In this study, we examine the question whether the popularity of portfolio insurance strategies can be explained using elements of behavioral finance. This is the novel path that our analysis takes. Recognizing that the optimality of portfolio insurance strategies depends on an investor's utility function, we analyze the return distribution of portfolio insurance strategies together with simple benchmark strategies using Tversky and Kahneman's (1992) cumulative prospect theory. In order to assess the attractiveness of portfolio insurance strategies from the perspective of a prospect theory investor, we start with Monte Carlo simulations and assume that continuously compounded stock market returns follow a Geometric Brownian motion. By calibrating the mean and volatility parameters of the return generating process, Monte Carlo simulations enable us to investigate the impact of changing stock market scenarios on cumulative prospect values. In a second step, we use German data and conduct historical simulations on a rolling windows basis. Historical simulation preserves the time series properties of the original return data. Both simulation approaches are applied on a step-by-step basis, where each module incorporates an additional feature of prospect theory. This procedure enables us to determine which components of prospect theory are responsible for the attractiveness of portfolio insurance strategies.

Our findings indicate that the stop-loss strategy, the synthetic put strategy, and the CPPI strategy provide returns that are more attractive for a prospect theory investor than the returns from various benchmark strategies. This main result is robust and shows up in different scenarios. However, it is not observable for the time invariant portfolio protection (TIPP) strategy, which is a variant of the CPPI strategy and exhibits a lower potential to participate from upward market movements compared to the other insurance strategies. Moreover, our results suggest that the cumulative prospect values of the synthetic put strategy are not very sensitive to estimation errors in the stock market volatility. In the spirit of Shefrin and Statman (1993) and Breuer and Perst (2007), our study also presents hints how a portfolio insurance product should be designed in order to meet a prospect theory investor's preferences as accurately as possible. Sensitivity analyses for all portfolio insurance strategies suggest that a higher protection level is more attractive for prospect theory investors than a lower one despite its reduced return potential. Moreover, given a maximum acceptable overnight or gap risk, a CPPI strategy should be implemented as aggressively as possible.

The remainder of this paper is structured as follows: Section 2 provides a discussion of the portfolio insurance strategies we use in our empirical analysis. Section 3 introduces the main elements of cumulative prospect theory. Section 4 discusses our results from Monte Carlo simulations and historical simulations. Section 5 addresses the question how a portfolio insurance product should be designed from the standpoint of a prospect theory investor. Section 6 concludes and discusses further implications for the asset management practice.

2. Alternative strategies to provide portfolio insurance

The main idea of portfolio insurance strategies is to offer participation from positive stock market movements, while simultaneously limiting potential losses to a pre-specified level or floor (e.g., a maximum loss of 0%, 5%, or 10% per year). Accordingly, the resulting return distribution becomes asymmetric and right-skewed. Various investment strategies that provide loss protection are suggested in the literature. Prominent examples (1) the stop-loss portfolio insurance strategy, (2) the synthetic put portfolio insurance strategy, (3) the constant proportion portfolio insurance (CPPI) strategy, and (4) the time invariant portfolio protection (TIPP) strategy. In this section, we provide a brief description of these strategies.

2.1. Stop-loss portfolio insurance strategy

The simplest way to protect a risky portfolio against losses is the stop-loss portfolio insurance strategy. In this strategy, the investor initially invests his total wealth (W_0) in the risky asset. This position is maintained as long as the market value of the risky position (W_t) exceeds the net present value (NPV) of the floor (F_T), which represents the minimum accepted portfolio value:

$$W_t > NPV(F_T). \tag{1}$$

If the market value of the portfolio reaches or drops below the discounted floor, hence if $W_t \leqslant NPV(F_T)$, all of the risky portfolio holdings are sold and invested in the risk-free asset. This position is held until the end of the investment horizon. If the interim portfolio value did not drop below $NPV(F_T)$, the investor's final wealth will never be lower than F_T. The stop-loss portfolio insurance strategy is easy to implement, it does not depend on any specific assumptions, and it also does not require estimating any model parameters (e.g., the stock market volatility). A disadvantage of this strategy is that the investor can no longer participate from any upward market movement once the portfolio has been shifted in the risk-free asset.[1]

2.2. Synthetic put portfolio insurance strategy

A second popular portfolio insurance strategy is the synthetic put strategy (Rubinstein and Leland, 1981). While a protective put strategy requires the availability of an adequate and liquid put option with the appropriate strike price and the desired time to maturity (Figlewski et al., 1993), the synthetic put strategy can be implemented in a fairly flexible manner. In its purest form

[1] In order to mitigate this deficiency, modified versions of the stop-loss strategy have been developed (e.g., Bird et al. (1988) and Bookstaber's (1985) multi-point stop-loss strategy).

the latter strategy uses the Black and Scholes (1973) option pricing formula to create a continuously adjusted synthetic European put option on the risky asset. Combining the purchase of a risky asset with the purchase of a put option on this asset (stock) is equivalent to buying a continuously-adjusted portfolio, which is a combination of the risky asset and the risk-free asset (cash). Pricing the put option with the Black and Scholes (1973) option pricing formula, the value of a portfolio that consists of a stock S plus a put P can be calculated as:

$$\begin{aligned} S+P &= S - S\cdot N(-d_1) + K\cdot e^{-rT}N(-d_2) \\ &= S\cdot[1-N(-d_1)] + K\cdot e^{-rT}N(-d_2) \\ &= S\cdot N(d_1) + K\cdot e^{-rT}N(-d_2), \end{aligned} \tag{2}$$

where K is the strike price, r the risk-free rate, and T the time to maturity. $N(\cdot)$ is the standard normal cumulative distribution function with d_1 and d_2 defined as:

$$d_1 = \frac{\ln(S/K) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}} \tag{3a}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}, \tag{3b}$$

where σ is the standard deviation of risky asset returns. In order to calculate the investment in the risky asset in the replicating portfolio, the delta of the portfolio in Eq. (2) is given as:

$$\frac{\partial(S+P)}{\partial S} = N(d_1). \tag{4}$$

The delta in Eq. (4) defines how much of the risky asset must be purchased in order to replicate the portfolio consisting of the risky asset and the put option. Multiplying the delta with the price of the risky asset S and dividing by the value of the portfolio in Eq. (2), the percentage allocations (w) in the risky asset and the risk-free asset are:

$$w_{\text{risky}} = \frac{S\cdot N(d_1)}{S\cdot N(d_1) + K\cdot e^{-rT}\cdot N(-d_2)} \tag{5a}$$

and

$$w_{\text{risk-free}} = 1 - w_{\text{risky}}. \tag{5b}$$

At the inception of a period, an investor who follows a synthetic put portfolio insurance strategy will invest in a portfolio of the risky asset and the risk-free bond. This strategy requires increasing (decreasing) the proportion of the risky asset in the portfolio if the price of the risky asset increases (decreases). In order to maintain a desired protection level, the strike price K must be set such that the following relationship holds:

$$K = \frac{F}{W_0}\cdot(S + P(K)), \tag{6}$$

where the ratio (F/W_0) is the percentage floor. The value of the put option $P(K)$ depends on the strike price itself, and hence the solution of Eq. (6) must be determined iteratively. The put's exercise price K will be higher than the floor because the put itself is costly.

Given the assumptions of the Black and Scholes (1973) framework, such as the absence of transaction costs, the portfolio must be readjusted on a continuous basis in order to maintain the desired protection level. However, transaction costs will incur with each portfolio adjustment, and hence a higher adjustment frequency leads to higher transaction costs. In order to incorporate this transaction costs effect, Leland (1985) and Boyle and Vorst (1992) suggest using the synthetic put portfolio insurance strategy with a modified volatility estimator:

$$\sigma_{\text{Leland}} = \sigma\sqrt{1 + \sqrt{\frac{2}{\pi}}\cdot\frac{k}{\sigma\sqrt{\Delta t}}} \tag{7a}$$

$$\sigma_{\text{Boyle/Vorst}} = \sigma\sqrt{1 + 2\cdot\frac{k}{\sigma\sqrt{\Delta t}}}, \tag{7b}$$

where k captures the round-trip transaction costs, and Δt denotes the length of the rebalancing interval. With these volatility adjustments, transaction costs as well as the corresponding rebalancing frequency are taken into account.

However, two crucial issues arise in a synthetic put portfolio insurance strategy. First, a synthetic put strategy is based on the assumptions of the Black and Scholes (1973) option pricing model. Among other assumptions, the model suggests that stock returns are normally distributed following a Geometric Brownian motion. Second, the synthetic put portfolio insurance strategy requires estimating the volatility of the asset whose value is to be secured (Bird et al., 1990; Zhu and Kavee, 1988). Accordingly, the quality of the protection strongly depends on the precision of this estimate (Rendleman and O'Brien, 1990).

2.3. Constant proportion portfolio insurance strategy

An alternative approach to portfolio insurance is the constant proportion portfolio insurance (CPPI), originally suggested by Black and Jones (1987, 1988). Because the CPPI strategy is not based on option pricing theory, many of the associated problems can be mitigated, and hence the practical implementation of the CPPI strategy is straightforward. Starting point is an investor's risk capital at time t, called the "cushion". The current cushion (C_t) represents the difference between the current wealth at time t (W_t) and the net present value (NPV) of the floor (F_T), hence:

$$C_t = W_t - NPV(F_T). \tag{8}$$

The exposure to the risky asset at time t, denoted as e_t, is calculated by multiplying this cushion with the multiplier, labeled m:

$$e_t = m\cdot C_t. \tag{9}$$

The remainder of the investor's wealth is invested in the risk-free asset. In principle, the multiplier m can be set to any value, but its choice has a strong economic meaning. The inverse of the multiplier (1/m) represents the maximum sudden loss in the risky asset that may occur such that the cushion is not fully depleted and the portfolio value does not fall below the discounted floor. For example, with a multiplier of $m = 5$, the risky asset can lose at most 20% (1/5 = 0.20) without violating the floor. However, when a sudden loss of over 20% occurs, the value of the portfolio falls below the promised minimum value ("gambler's ruin"). In commercial applications, it is necessary to permanently control the optimal exposure to the risky asset, and hence portfolio shifts need to be executed immediately. In most cases a trading filter is used, implying that only significant up and down market movements cause shifts in the portfolio structure and irrelevant side market movements are filtered out. Although the maintenance of the floor will be controlled on an intraday basis, the risk still exists over night when the portfolio manager cannot react immediately to extreme market losses (overnight risk or gap risk). In spite of this small residual risk, which can partly be controlled by choosing an appropriate multiplier m, the CPPI strategy can be classified as an absolute protection strategy with a strictly lower limit for the portfolio value.[2]

[2] There exist other portfolio insurance strategies, which do not strictly guarantee a specific portfolio value (floor) but only with a specified probability. Strategies based on the Value-at-Risk (VaR) are discussed in Jiang et al. (2009), Herold et al. (2005) and Herold et al. (2007).

An unconstrained CPPI strategy can lead to short positions in the risk-free asset (when the price of the risky asset is high) or in the risky asset (when the price of the risky asset is low). In commercial applications the CPPI strategy is usually implemented such that the holding of the risky asset varies between 0% and 100% of the investment sum. This implies that short-sales and leverage are ruled out. In this case, Eq. (9) is modified as follows (Benninga, 1990; Do, 2002; Annaert et al., 2009):

$$e_t = \max[\min(m \cdot C_t, W_t), 0]. \tag{10}$$

The constrained CPPI strategy can be implemented by shifting between the stock market and cash as dictated by Eq. (10). Therefore, in our simulations we use this rule to implement the CPPI strategy in each simulation path.

All portfolio insurance strategies pursue the goal to secure the initial wealth up to a pre-specified protection level or floor. However, Estep and Kritzman (1988) argue that investors will not only be interested in a protection of their initial wealth, but also in the protection of any interim capital gains. In order to achieve this additional effect, they suggest a modification of the CPPI strategy, which they call the "time invariant portfolio protection" (TIPP) strategy. While the CPPI strategy operates with a fixed floor (which is the initial wealth multiplied by the percentage floor), the floor of the TIPP strategy is ratchet up if the value of the portfolio increases. Specifically, after choosing the initial floor and the multiplier, this strategy requires the following steps (Estep and Kritzman, 1988):

1. Calculation of the actual portfolio value (stocks plus cash).
2. Multiplication of this portfolio value by the floor percentage.
3. If the result in step 2 is greater than the previous floor, this level becomes the new floor; otherwise the old floor is kept.
4. Application of the CPPI strategy as dictated by Eqs. (8)–(10).

While the principle idea behind the TIPP strategy seems attractive, Choie and Seff (1989) argue that this strategy suffers from a major shortcoming. As in the traditional CPPI strategy, the TIPP strategy transfers all holdings of the risky asset in an irreversible manner to the risk-free asset once the floor has been reached. Accordingly, the TIPP strategy cannot participate from any subsequent upward market movements. However, because of the continuous "ratcheting up" of the floor to the highest portfolio value, the likelihood that the portfolio value reaches or falls below the prevailing floor increases, and hence the TIPP strategy will more often end up fully invested in the risk-free asset. The overall net effect on the mean return of the TIPP strategy compared to the standard CPPI strategy is not clear ex ante, but our simulation framework is able to provide detailed insights.

3. Prospect theory versus expected utility theory

Expected utility theory is based on three principles: (1) the overall expected utility of a choice is equal to the sum of the probability weighted utilities of all possible outcomes; (2) a choice is acceptable if it adds value to the existing asset portfolio; (3) all investors are strictly risk-averse. Standard finance is based on the assumption that investors behave rationally and take investment decisions that optimize expected utility. Accordingly, expected utility theory is a normative theory. In our simulation analysis, however, we assume a prospect theory investor. Prospect theory starts from empirical evidence to describe how individuals choose between alternatives that involve risk. Several behavioral phenomena have been discussed in the literature to describe how investors evaluate potential gains and losses:

- Prospect theory investors evaluate their choices in terms of the potential gains and losses relative to investor specific reference points, a phenomenon which refers to the wider concept of framing. This is in contrast to expected utility theory, where investors evaluate their choices in terms of total expected wealth.
- While standard investors are always risk-averse, prospect theory investors are risk-averse in the domain of gains but risk-seeking in the domain of losses. This assumption implies an S-shaped value function, which passes through the reference point and is concave over gains and convex over losses. The value function replaces the standard utility function.
- Prospect theory investors exhibit loss aversion. Given the same variation in absolute value away from the reference point, there is a bigger impact of losses than of gains. Gains and losses of the same amount are valued in an asymmetric way, and investors care more about potential losses than potential gains.[3]
- In contrast to expected utility theory, prospect theory investors weight the probabilities of various outcomes instead of using the statistical probabilities. This approach allows incorporating the empirical observation that investors overweight events with low probability of occurrence, but underweight "average" events.

Expected utility theory assumes that investors are risk-averse with a concave value (utility) function. In contrast, prospect theory expresses outcomes as deviations (positive and negative) from a reference point (Δx) and suggests an S-shaped value function, with the curve being concave for gains and convex for losses. In addition, investor's response to a loss is more extreme than the response to a gain, implying that the value function is steeper for losses than for gains. To capture these properties when evaluating a stochastic outcome Δx, Tversky and Kahneman (1992) suggest a two-part valuation function $v(\Delta x)$:

$$v(\Delta x) = \begin{cases} (\Delta x)^{\alpha} & \Delta x \geqslant 0 \\ -\lambda \cdot (-\Delta x)^{\beta} & \Delta x < 0 \end{cases}, \tag{11}$$

with $\alpha \approx \beta \approx 0.88$ and $\lambda \approx 2.25$. The S-shaped value function in Eq. (11) is concave in the domain of gains (which implies risk-aversion) and convex in the domain of losses (which implies risk-seeking). The parameter λ captures loss aversion, assuming that investors consider losses more than twice as important as gains. Prelec (2000) argues that the value function in Eq. (11) describes by far the most popular way to estimate money value. Moreover, instead of weighting the subjective values according to Eq. (11) with their statistical probabilities, we use the probability weighting function suggested by Lattimore et al. (1992):

$$w_{\delta,\gamma}(p) := \frac{\delta \cdot p^{\gamma}}{\delta \cdot p^{\gamma} + (1-p)^{\gamma}} := \begin{cases} w^{+}(p) := \frac{\delta^{+} \cdot p^{\gamma^{+}}}{\delta^{+} \cdot p^{\gamma^{+}} + (1-p)^{\gamma^{+}}} & \Delta x \geqslant 0 \\ w^{-}(p) := \frac{\delta^{-} \cdot p^{\gamma^{-}}}{\delta^{-} \cdot p^{\gamma^{-}} + (1-p)^{\gamma^{-}}} & \Delta x < 0 \end{cases}. \tag{12}$$

The probability weighting function in Eq. (12) allows to distinguish between two essential features based on the following parameters: (1) the parameter γ mainly controls curvature, and (2) the parameter δ mainly controls elevation.[4] These two parameters incorporate the experimental observation that prospect theory investors tend

[3] Moreover, Berkelaar and Kouwenberg (2009) show that loss aversion is time-varying. In very good states loss-averse investors become gradually less risk-averse as wealth rises above their reference point, pushing up equity prices. In contrast, when wealth drops below the reference point investors become risk-seeking and demand for stocks increases, eventually leading to a forced sell-off and a stock market bust in bad states.

[4] Ingersoll (2008) discusses shortcomings of the single-parameter probability weighting function suggested by Tversky and Kahneman (1992).

to overweight small probability events. Based on the empirical results in Abdellaoui (2000), we choose $\delta^+ = 0.65$, $\delta^- = 0.84$, $\gamma^+ = 0.6$, and $\gamma^- = 0.65$.[5]

The original version of the prospect theory (Kahneman and Tversky, 1979) suffers from potential violations of first-order stochastic dominance, implying that one prospect might be preferred even if it yields a worse outcome with probability one. In order to avoid a violation of first order stochastic dominance, we apply Tversky and Kahneman's (1992) cumulative prospect theory. While single probabilities are weighted in prospect theory, the cumulative probabilities are weighted in cumulative prospect theory:

$$\pi_i := \begin{cases} \pi_i^- = w^-(p_1 + \cdots + p_i) - w^-(p_1 + \cdots + p_{i-1}) \\ \pi_i^+ = w^+(p_i + \cdots + p_N) - w^+(p_{i+1} + \cdots + p_N) \end{cases}, \tag{13}$$

where i denotes the outcomes Δx_i (with $i = 1,\ldots,N$), which are assumed to be sorted in ascending order. After weighting the cumulative probabilities in Eq. (12), the differences between neighboring probability weightings (π_i) are computed. As described in Tversky and Kahneman (1992), the decision weight π_i^-, associated with a negative outcome, is the difference between the weighted cumulative probabilities (w-values) of the events "the outcome is at least as bad as Δx_i", and "the outcome is strictly worse than Δx_i". Similarly, the decision weight π_i^+, associated with a positive outcome, is the difference between the w-values of the events "the outcome is at least as good as Δx_i", and "the outcome is strictly better than Δx_i".[6] Given these decision weights (π_i), the cumulative prospect value (CPV) of a strategy is:

$$CPV(\text{Strategy}) = \sum_{i=1}^{N} \pi_i \cdot v(\Delta x_i). \tag{14}$$

In our simulation analyses, we apply Eqs. (11)–(14) on a step-by-step basis. This step-wise procedure allows us to determine which features of prospect theory are responsible for the attractiveness of portfolio insurance strategies for a prospect theory investor compared to other (benchmark) investment strategies.

4. Simulation analysis

This section presents our simulation results. We start with Monte Carlo simulations in Section 4.1 and proceed with historical simulations in Section 4.2. We also present the results from a battery of robustness tests.

4.1. Monte Carlo simulations

4.1.1. Simulation design

We start our analysis by running Monte Carlo simulations. In this setup, we assume idealized stock markets in the sense that observable phenomena such as autocorrelation, skewness, and fat tails are neglected. However, using different choices of the stochastic parameters, we are able to compare different economic scenarios. When modeling stock market returns, there are two important parameters: (1) the equity risk premium and (2) the stock market volatility. In order to analyze the influence of these factors in a systematic manner, we distinguish between four states of nature, which are summarized in Table 1.

First, we assume a high and a low equity risk premium. Dimson et al. (2006) report that the mean annual equity risk premium for developed stock markets was approximately 7% between 1900 and 2005. We take this long-run average value to represent the high risk premium state. Dimson et al. (2006) further elaborate why the future equity risk premium will be lower, and they estimate the expected excess return to be around 4.5% per year. We use this conservative estimate to represent the low risk premium state. Second, we distinguish between a high and a low stock market volatility state. We take a "normal" stock market return volatility to represent the low volatility state. According to Dimson et al. (2006), the long-run stock return volatility was roughly 20% per year. Benninga (1990) and Figlewski et al. (1993) also use this value in their simulation studies. In contrast, we use a stock market volatility of 30% per year in the high volatility state. In addition to turbulent market environments, volatility of this magnitude can be justified if the focus is on small cap stocks. Following Arnott and Bernstein (2002), the rate of return on the risk-free asset is fixed at 4.5%. Adding this value to the equity risk premium, we have an expected annual stock market return of 9% in the low risk premium state and of 11.5% in the high risk premium state.

Table 1
The four stock market scenarios.

	Risk premium (simple returns)	
	Low	High
Volatility		
Low (normal)	Scenario 1: • Risk premium: 4.5% • Volatility: 20%	Scenario 2: • Risk premium: 7% • Volatility: 20%
High	Scenario 3: • Risk premium: 4.5% • Volatility: 30%	Scenario 4: • Risk premium: 7% • Volatility: 30%

Our Monte Carlo simulations generate continuously compounded stock market returns, denoted as $d(\ln S)$, on the basis of a Geometric Brownian motion (Hull, 2008)[7]:

$$d(\ln S) = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dz. \tag{15}$$

In order to specify the mean (μ) and volatility (σ) parameters, we transform the expected returns and standard deviations shown in Table 1 (based on simple returns) into their corresponding continuously compounded counterparts. The term dz represents a Wiener process that describes the evolution of a normally distributed variable. The stochastic process for stock returns in Eq. (15) is consistent with the assumptions of the Black and Scholes (1973) option pricing model, and hence with the synthetic put strategy.

Because many institutional and private investors use a one year investment horizon (Benartzi and Thaler, 1995), we also focus on this investment horizon in our simulation analysis. Using the stochastic process in Eq. (15), we simulate 250 daily stock market returns and apply the various portfolio insurance strategies. We take 10 basis points round-trip transaction costs into account.[8] In order to avoid portfolio shifts that are triggered by trendless market movements, we implement the CPPI strategy, the TIPP strategy, and the synthetic put strategy with a trading filter. Portfolio shifts are only executed when the stock market moves by more than 2%.[9] Moreover,

[5] Gurevich et al. (2009) confirm that the shape of the value function and the probability weighting function estimated from option data is in line with theory.

[6] See Tversky and Kahneman (1992), p. 301. They describe the special properties of the weighting function in Eq. (13) in more detail and provide a simple numerical example.

[7] Most studies that analyze portfolio insurance strategies based on Monte Carlo simulations use this process for the generation of stock market returns (Benninga, 1990; Bird et al., 1990; Zhu and Kavee, 1988; Figlewski et al., 1993; Cesari and Cremonini, 2003; Herold et al., 2007).

[8] The same transaction costs are used in Herold et al. (2007). They argue that transaction costs of 10 basis points are a conservative estimate when trades are executed with futures.

[9] The same trading filter is used in Do and Faff (2004). They explain their choice on the basis of observed daily index movements. A comparable trading filter is used in Cesari and Cremonini (2003); they only trade if the stock market moves more than 2.5% in one direction.

we implement the CPPI strategy and the TIPP strategy with a base case multiplier of $m = 5$, which is a commonly used value in commercial applications (Herold et al., 2007). The synthetic put strategy uses the "true" volatility (i.e., 20% or 30%), but it is modified as suggested by Boyle and Vorst (1992).[10] Moreover, we implement the portfolio insurance strategies with a protection level of 100% (full capital guarantee) and compare their performance with a buy-and-hold stock market investment (e.g., the simple return of a passive stock market fund) and the risk-free rate (e.g., the simple return of a money market fund). As another benchmark, we contrast our simulation results with a balanced strategy, where 50% of the portfolio is invested in the stock market and 50% in cash over the one year horizon (without monthly rebalancing). In order to derive the full distribution of outcomes, we perform 100,000 simulation runs. Repeated simulations reveal that our results are very stable.

The simulation setup allows us to examine whether the popularity of portfolio insurance strategies can be explained within the framework of prospect theory. Prospect theory suggests that investors evaluate their opportunities relative to a reference point. It seems plausible to assume that for many investors the purchasing price of an asset is the relevant reference point. Investors assess their investment opportunities in terms of absolute returns, where positive returns represent gains and negative returns losses (Kahneman and Riepe, 1998; Fisher and Statman, 1999). Therefore, we use simple returns (and hence a reference point of zero) to compute prospect values based on the valuation function in Eq. (11). In order to assess the impact of the various elements of prospect theory (e.g., the S-shaped value function, loss aversion, or probability weighting), we implement our simulations on a step-by-step basis. This modular simulation design enables us to examine the contribution of the different features of prospect theory for the attractiveness of portfolio insurance strategies:

1. In the first step, we use the valuation function in Eq. (11), set the parameter $\lambda = 1$, and omit probability weightings. Accordingly, we take the simple mean of the simulated prospect values. This most basic setup incorporates the S-shaped valuation function with risk-aversion in the domain of gains and risk-seeking in the domain of losses. However, it does not account for loss aversion.
2. In the second step, we use the valuation function in Eq. (11) and set the loss aversion parameter $\lambda = 2.25$. Again, we omit using a probability weighting. This extended setup is consistent with prospect theory and captures the simultaneous risk-avoiding and risk-seeking behavior together with loss aversion.
3. In the third step, we incorporate the elements of cumulative prospect theory as shown in Eqs. (12)–(14). Any observable deviations of the mean cumulative prospect values from the simulation results in the previous steps must be attributed to the impact of the cumulative probability weightings.

4.1.2. Main simulation results

Table 2 shows our simulation results for all portfolio insurance strategies with a 100% protection level together with the three benchmark strategies. Panel A presents the results for scenario 1 with a conservative equity risk premium of 4.5% and a stock market volatility of 20% per year. As one would expect, the stock market investment exhibits a higher mean return than all four portfolio insurance strategies due to the risk premium effect (e.g., 9.03% vs. 6.22% for the stop-loss strategy). However, the standard deviations of the portfolio insurance strategies are significantly lower than the volatility of the passive stock market strategy.[11] This is particularly true for the TIPP strategy and the CPPI strategy (with a volatility of 2.24% and 5.88%, respectively). Based on the Sharpe-ratio, both the passive stock market strategy and the balanced strategy dominate all four portfolio insurance strategies. However, the most important observation is that these findings reverse when the analysis is based on cumulative prospect values. While the passive stock market investment exhibits a cumulative prospect value of −0.02, the CPPI strategy delivers a cumulative prospect value of 4.54, the TIPP strategy of 3.54, the stop-loss strategy of 4.65, and the synthetic put strategy of 4.28. We conduct a paired t-test to validate that the cumulative prospect values of the portfolio insurance strategies are significantly different from the benchmark strategy with the highest cumulative prospect value.[12] In Panel A, the cash investment boasts the highest cumulative prospect value of 3.76 among all benchmark strategies. This value is higher than the cumulative prospect value of the TIPP strategy, but clearly lower than the corresponding values of the CPPI stategy, the stop-loss strategy, and the synthetic put strategy. All differences are statistically significant at the 1% level. Therefore, at least from the standpoint of a prospect theory investor, a portfolio insurance strategy with a 100% protection level (with exception of the TIPP strategy) seems to be an attractive investment strategy.

In order to provide more detailed insights, we compare the mean prospect values that are derived with loss aversion parameters $\lambda = 1$ and $\lambda = 2.25$.[13] While the passive stock market investment exhibits a significantly higher mean prospect value than all portfolio insurance strategies for $\lambda = 1$ (e.g., 5.92 vs. 4.27 for the CPPI strategy), the opposite ranking of the strategies is observed for $\lambda = 2.25$. This finding suggests that the S-shaped utility function alone is not able to explain the superiority of portfolio insurance strategies. However, if loss aversion is incorporated as an additional feature of prospect theory, all four portfolio insurance strategies deliver a higher mean prospect value than the passive stock market strategy (e.g., 4.27 vs. 2.41 for the CPPI strategy).[14] Accordingly, consistent with the finding by Hwang and Satchell (2010) that investors are more loss-averse than usually assumed, loss aversion is one explanation for the attractiveness of portfolio insurance strategies. Observing that the difference between the cumulative prospect values of the portfolio insurance strategies and the passive stock market investment is higher than the difference between the mean prospect values with $\lambda = 2.25$ (e.g., 4.54 vs. −0.02 compared to 4.27 vs. 2.41 for the CPPI strategy), the probability weighting scheme further contributes to the attractiveness of protection strategies. An explanation for this finding is that extremely adverse states, whose negative prospect values cannot be offset by the positive risk premium effect during the one year investment horizon,

[10] We also run the simulations using the original (unmodified) volatility and the modification suggested by Leland (1985). However, the simulation results are not sensitive to these changes.

[11] We recognize that the standard deviation is not the preferable risk measure when analyzing investment strategies with an asymmetric return distribution. Presumably, downside risk measures (e.g., lower partial moments or Value-at-Risk) are more appropriate. However, our analysis focuses on prospect values.

[12] A paired t-test is frequently used to test if the performance or risk measures of investment strategies are significantly different. For example, Annaert et al. (2009) use a paired t-test to test the differences of expected shortfall measures. We recognize that there exist other hypothesis tests that are potentially more powerful, such as tests that take the whole distribution into account (Linton et al., 2005).

[13] We again use a paired t-test to test if the mean prospect values of the various protection strategies are different from the respective benchmark strategy with the hightest mean prospect value. For example, in scenario 1 (Panel A) the stock market investment exhibits the highest mean prospect value (5.92) for $\lambda = 1$. However, the money market investment boasts the highest mean prospect value (3.76) for $\lambda = 2.25$.

[14] For the CPPI strategy and the TIPP strategy the mean prospect values are identical for $\lambda = 1$ and $\lambda = 2.25$. This indicates that the floor (100%) is not violated, and hence the value of the loss aversion parameter is not relevant. However, this result cannot be observed for the stop-loss strategy and the synthetic put strategy.

Table 2
Base case Monte Carlo simulation results.

	CPPI	TIPP	Stop-loss	Synthetic put	Stock market	Cash	50:50 B&H
Panel A: Expected return = 9%, volatility = 20%							
Mean return p.a. (%)	5.47	4.89	6.22	6.20	9.03	4.50	6.76
Volatility p.a. (%)	5.88	2.24	14.72	10.75	19.90	0.00	9.95
Sharpe-ratio	0.16	0.17	0.12	0.16	0.23	–	0.23
Mean prospect value (λ = 1.0)	4.27**	4.00**	3.90**	4.33**	5.92	3.76	4.84
Mean prospect value (λ = 2.25)	4.27**	4.00**	3.18**	3.93**	2.41	3.76	3.60
Cumulative prospect value	4.54**	3.54**	4.65**	4.28**	−0.02	3.76	2.00
Panel B: Expected return = 11.5%, volatility = 20%							
Mean return p.a. (%)	6.10	5.13	7.51	7.40	11.46	4.50	7.98
Volatility p.a. (%)	6.25	2.25	15.79	11.47	19.82	0.00	9.91
Sharpe-ratio	0.26	0.28	0.19	0.25	0.35	–	0.35
Mean prospect value (λ = 1.0)	4.71**	4.17**	4.75**	5.16**	7.55	3.76	5.72
Mean prospect value (λ = 2.25)	4.71*	4.17**	4.08**	4.81**	4.74	3.76	4.75
Cumulative prospect value	4.93**	3.70**	5.35**	4.93**	1.97	3.76	3.02
Panel C: Expected return = 9%, volatility = 30%							
Mean return p.a. (%)	5.31	4.78	5.73	5.63	8.96	4.50	6.73
Volatility p.a. (%)	9.65	2.84	18.95	13.23	29.88	0.00	14.94
Sharpe-ratio	0.08	0.10	0.06	0.09	0.15	–	0.15
Mean prospect value (λ = 1.0)	3.97**	3.88**	3.29**	3.73**	5.44	3.76	4.53
Mean prospect value (λ = 2.25)	3.97**	3.88**	2.20**	3.32**	−0.85	3.76	1.91
Cumulative prospect value	5.20**	3.44**	4.86**	4.79**	−2.82	3.76	0.51
Panel D: Expected return = 11.5%, volatility = 30%							
Mean return p.a. (%)	5.96	4.99	6.63	6.56	11.54	4.50	8.02
Volatility p.a. (%)	10.27	2.84	20.09	14.12	29.82	0.00	14.91
Sharpe-ratio	0.14	0.17	0.11	0.15	0.24	–	0.24
Mean prospect value (λ = 1.0)	4.41**	4.04**	3.85**	4.35**	7.11	3.76	5.43
Mean prospect value (λ = 2.25)	4.41**	4.04**	2.81**	3.97**	1.70	3.76	3.23
Cumulative prospect value	5.62**	3.57**	5.46**	5.35**	−0.70	3.76	1.61

This table shows the results from our base case Monte Carlo simulations for portfolio insurance strategies. Continuously compounded stock market returns are generated using a Geometric Brownian motion. The assumptions for expected returns and standard deviations in Panels A–D are taken from the four stock market scenarios shown in Table 1. The risk-free rate is fixed at 4.5%. We simulate 250 daily stock market returns and implement the portfolio insurance strategies with 10 basis points round-trip transaction costs, a trading filter of 2%, and a protection level of 100%. We perform 100,000 simulation runs in order to derive the full distribution of prospect values for the portfolio insurance strategies and the benchmark strategies (a passive stock market fund, a money market fund (cash), and a balanced buy-and-hold strategy (50:50 B&H) without monthly rebalancing). The null hypothesis in the paired t-test is that the (mean or cumulative) prospect value of a portfolio insurance strategy is equal to that of the best corresponding benchmark strategy.

* The test statistic is significant at the 5% level.

** The test statistic is significant at the 1% level.

receive higher probability weights. Similar results can be observed for the balanced strategy.

The comparison with the money market investment reveals additional insights. Cash earns an annual return of 4.5% without any loss potential, and hence one would expect that it is attractive for a prospect theory investor. In fact, with a cumulative prospect value of 3.76, the money market investment dominates the passive stock market strategy and the balanced strategy (with cumulative prospect values of −0.02 and 2.00, respectively). However, its cumulative prospect value is lower than that of the CPPI strategy (3.76 vs. 4.54), the stop-loss strategy (3.76 vs. 4.65), and the synthetic put strategy (3.76 vs. 4.28). Again, this result is not observable for the TIPP strategy. The TIPP strategy exhibits a significantly lower cumulative prospect value than the money market investment (3.54 vs. 3.76). An explanation for this observation could be that the CPPI strategy, the stop-loss strategy, and the synthetic put strategy protect an investor's initial wealth, while the TIPP strategy also attempts to protect all interim capital gains. Specifically, in the TIPP strategy the initial floor is dynamically ratchet up whenever the investor earns capital gains. An implication of a higher floor is that the current wealth more often reaches or drops below this level, in which case all holdings of the risky asset are shifted into the risk-free asset. The cash position is then held until the end of the investment horizon, and hence a TIPP strategy investor cannot participate from any subsequent upward market movements. This higher frequency of ending up in the risk-free asset (compared to the CPPI strategy) explains why the TIPP strategy generates the lowest mean return of all four portfolio insurance strategies. In fact, with an annual mean return of 4.89%, this protection strategy exhibits only a slightly higher return potential than the cash investment (4.50%). Given that the TIPP strategy avoids losses at least as good as the simpler CPPI strategy, the inferiority of this strategy must be a result of its reduced upside participation.

The CPPI strategy dominates the TIPP strategy in terms of mean prospect values (4.27 vs. 4.00), and the difference becomes more pronounced based on cumulative prospect values (4.54 vs. 3.54). This finding indicates that probability weighting (as shown in Eqs. (12) and (13)) makes the CPPI strategy (as well as the stop-loss strategy and the synthetic put strategy) more attractive and the TIPP strategy less attractive for prospect theory investors. Since the probability weighting scheme puts high weights on extreme but rare events, an explanation is the insufficient participation of the TIPP strategy in upward market movements.

Panel B displays the simulation results for the high return state with an equity risk premium of 7% per year (hence a total return of 11.5%) in combination with a volatility of 20% per year. Our major results remain valid in this second scenario. Although the cumulative prospect values of all four portfolio insurance strategies are still higher than the corresponding value of the passive stock market investment, the differences shrink compared to Panel A. More important, in most cases the combination of an S-shaped utility function and loss aversion is not sufficient to explain our findings. The mean prospect value with a loss aversion parameter λ = 2.25 is 4.71 for the CPPI strategy, 4.17 for the TIPP strategy, and 4.08 for the stop-loss strategy, while the corresponding value for a stock market investment is 4.74. Therefore, the superiority of protection strategies over the stock market investment is dependent on the probability weighting scheme.[15] As in Panel A, the TIPP strategy is the only strategy with a lower cumulative prospect value than the money market investment.

Panel C shows the simulation results when the equity risk premium is low (with a total return of 9% per year) and the volatility is high (with a standard deviation of 30% per year). In this third scenario all portfolio insurance strategies dominate the stock market investment and the balanced strategy in terms of their cumulative prospect values. The cumulative prospect value of the stock market strategy even becomes negative (−2.82). An explanation for this observation is that a higher return volatility causes large gains (positive returns) but also large losses (negative returns). Because of an overweighting of negative returns relative to positive returns, a symmetric return effect leads to a lower cumulative prospect value for the stock market investment. A comparison of the mean prospect values for $\lambda = 1.0$ and for $\lambda = 2.25$ supports this hypothesis. The mean prospect value of the passive stock market investment decreases from 5.44 (without loss aversion) to −0.85 (with loss aversion). In a portfolio insurance strategy large positive and large negative stock market returns (due to higher volatility) have different effects. On the one hand, a protection strategy avoids large negative stock market returns, and hence they cannot exert a negative effect on the cumulative prospect value. On the other hand, large positive stock market returns have a positive impact on the cumulative prospect value. This property of protection strategies – to avoid the downside potential while maintaining some upside potential – explains their favorable impact on the cumulative prospect value. Again, the TIPP strategy is the only strategy with a lower cumulative prospect value than the money market investment.

Finally, the simulation results in Panel D (with a high equity risk premium and a high volatility) also confirm our main findings. In this scenario, the cumulative prospect value of the passive stock market investment is still negative (−0.70). Compared to Panel C, however, the increase of the expected return from 9% to 11.5% per year (given a high stock market volatility of 30% per year) in Panel D leads to an increase of the cumulative prospect value for all four portfolio insurance strategies.

Overall, our Monte Carlo simulation results reveal that the CPPI strategy, the stop-loss strategy, and the synthetic put strategy significantly dominate all three benchmark strategies in all four stock market scenarios in terms of their cumulative prospect values, and hence they are an attractive investment strategy for a prospect theory investor. In contrast, the TIPP strategy exhibits a cumulative prospect value that makes it inferior compared to the money market investment in all four stock market scenarios.

4.1.3. Robustness tests

An immediate question is whether our simulation analysis uses adequate benchmark strategies. We cannot rule out that there exist other benchmark strategies which provide higher cumulative prospect values than a money market investment (which is the benchmark strategy with the highest cumulative prospect value in all four scenarios). As a robustness test, we construct alternative benchmark strategies in the spirit of Sharpe's (1992) style analysis. We measure the average stock market allocation of each protection strategy in each single simulation run and compute the mean value over all 100,000 simulation runs. These mean values are dependent on the stock market development, and hence we obtain for each protection strategy and for each stock market scenario an average stock market allocation. Based on these average stock market exposures, we construct alternative benchmark strategies similar to our balanced strategy. These mixed stock-money market benchmark strategies consist of a $x\%$ stock market allocation and a $(1 - x\%)$ cash position, where the fixed weight x denotes the strategy- and scenario-specific average stock market exposure. We again consider buy-and-hold strategies without monthly rebalancing. The cumulative prospect values of these alternative stock-money market strategies together with the strategy- and scenario-specific average stock market allocations are shown in Appendix A. With only one exception all additional benchmark strategies exhibit significantly lower cumulative prospect values than the money market investment, which remains the dominating benchmark strategy.[16]

In another battery of robustness tests, we rerun our Monte Carlo simulations with several modifications. In a first step, in Panel A of Table 3 we reduce the risk-free rate from 4.5% to 3.5%. As one would expect, the attractiveness of all portfolio insurance strategies decreases. Nevertheless, the dominance of the CPPI strategy, the stop-loss strategy, and the synthetic put strategy against the money market investment (as the most attractive benchmark strategy) in terms of cumulative prospect values remains unchanged in all four stock market scenarios. In a second step, in Panel B of Table 3 we increase the risk-free rate from 4.5% to 5.5%. All strategies become more attractive from the perspective of a prospect theory investor, but the CPPI strategy, the stop-loss strategy, and the synthetic put strategy again dominate the money market investment.

In order to analyze the dependence of our simulation results on the choice of the probability weighting scheme, we use the alternative probability weighting parameters suggested by Abdellaoui et al. (2005). While their probability weighting function is identical to that in Eq. (12), they choose the parameters δ and γ slightly different from the values that we use in our base case analysis (which are chosen according to Abdellaoui, 2000). In results not shown, the CPPI strategy, the stop-loss strategy, and the synthetic put strategy again exhibit higher cumulative prospect values than the three benchmark strategies in all four stock market scenarios. Moreover, in this framework even the TIPP strategy exhibits higher cumulative prospect values than the money market investment.

4.2. Historical simulations

Monte Carlo simulations have the advantage that they allow to derive a distribution under different economic scenarios. However, the stochastic process for stock returns that is used in Section 4.1 is only a limited model of financial markets. For example, stock market returns are characterized by short-term autocorrelation and long-term mean reversion (Poterba and Summers, 1988). Stock market returns also deviate from the normal distribution model with constant volatility (homoskedasticity). Specifically, stock market returns are often heteroskedastic and left-skewed, and they exhibit fat tails (Annaert et al., 2009; Kapetanios, 2009). Another simplification in our analysis is the assumption of a constant risk-free rate. In order to capture these effects, we perform a historical simulation using financial markets data. Specifically, we use daily return data for the German stock market index DAX

[15] In a slightly different context, Dierkes et al. (2010) also document that probability weighting is an important factor for portfolio insurance strategies' attractiveness.

[16] The only exception is the benchmark strategy, whose stock market allocation corresponds to the average stock market allocation of the TIPP strategy in scenario 2 (Panel B in Appendix A). This strategy exhibits a cumulative prospect value of 3.83 compared to 3.76 for the money market investment. However, when we compare this value with the cumulative prospect values of our protection strategies (Panel B in Table 2), we observe a dominance of the CPPI, the stop-loss, and the synthetic put strategy against this benchmark strategy.

Table 3
Robustness tests for Monte Carlo simulations.

	CPPI	TIPP	Stop-loss	Synthetic put	Stock market	Cash	50:50 B&H
Panel A: Risk-free rate = 3.5%							
Exp. return = 9%, volatility = 20%	3.82**	2.88**	4.15**	3.80**	−0.12	3.01	1.51
Exp. return = 11.5%, volatility = 20%	4.17**	3.02	4.85**	4.47**	1.94	3.01	2.59
Exp. return = 9%, volatility = 30%	4.44**	2.79**	4.32**	4.22**	−2.91	3.01	0.01
Exp. return = 11.5%, volatility = 30%	4.79**	2.89**	4.82**	4.72**	−0.76	3.01	1.14
Panel B: Risk-free rate = 5.5%							
Exp. return = 9%, volatility = 20%	5.18**	4.15**	5.05**	4.67*	−0.02	4.48	2.44
Exp. return = 11.5%, volatility = 20%	5.61**	4.34**	5.76**	5.37**	2.00	4.48	3.46
Exp. return = 9%, volatility = 30%	5.82**	4.03**	5.37**	5.25**	−2.89	4.48	0.91
Exp. return = 11.5%, volatility = 30%	6.37**	4.20**	6.05**	5.89**	−0.67	4.48	2.06

This table shows the results from robustness tests of our Monte Carlo simulations for portfolio insurance strategies. The entries exhibit cumulative prospect values. Continuously compounded stock market returns are generated using a Geometric Brownian motion. The assumptions for expected returns and standard deviations in Panels A and B are taken from the four stock market scenarios presented in Table 1. The risk-free rate is 3.5% in Panel A and 5.5% in Panel B. We simulate 250 daily stock market returns and implement the portfolio insurance strategies with 10 basis points round-trip transaction costs, a trading filter of 2%, and a protection level of 100%. We perform 100,000 simulation runs in order to derive the full distribution of prospect values for the portfolio insurance strategies and three benchmark investment strategies (a passive stock market fund, a money market fund (cash), and a balanced buy-and-hold strategy (50:50 B&H) without monthly rebalancing). The null hypothesis in the paired *t*-test is that the cumulative prospect value of a portfolio insurance strategy is equal to that of the money market investment (which is the benchmark strategy with the highest cumulative prospect value).
* The test statistic is significant at the 5% level.
** The test statistic is significant at the 1% level.

Table 4
Descriptive statistics of German stock and money market returns.

	Stock market (DAX) (%)	Risk-free rate (%)
Mean annual return	10.62	5.14
Annual volatility	24.36	2.50

This table shows descriptive statistics of the German stock and money markets based on discrete annual returns over the period from January 1980 to October 2009 (including 7778 daily returns).

(Deutscher Aktienindex) and money market rates from the German Bundesbank over the period from January 1980 to October 2009.[17]

Table 4 shows descriptive statistics of stock market returns and money market rates. The annual mean return of the German stock market is 10.62%; it falls between 9% and 11.5%, which are the expected stock market returns in the scenarios presented in Table 1. The historical volatility of 24.36% per year also falls between our high (30%) and low (20%) volatility state assumptions.

Table 5 presents the statistical properties of daily continuously compounded DAX returns. Panel A shows the autocorrelations of daily DAX returns. Although all values are close to zero, a Ljung–Box test up to lag 5 indicates statistically significant autocorrelations. Moreover, we conduct Engle's (1982) Lagrange multiplier test to detect heteroskedasticity. The results in Panel B indicate heteroskedasticity effects in daily DAX returns at lag 1 and up to lag 3. Finally, the Jarque–Bera test statistic in Panel C reveals that the null hypothesis of a normal distribution must be rejected. The negative value for the skewness (−0.32) indicates that the return distribution is left-skewed. Moreover, the value for the kurtosis (10.51) is significantly larger than 3, which is a hint for fat tails.

In our historical simulation, we use 250 subsequently following daily stock and money market returns on a rolling windows basis (moving the window forward by one day) and implement our portfolio insurance and benchmark strategies. This approach delivers 7529 overlapping yearly performance data for each investment strategy, and hence it uses the available data in a most efficient way. Most important, it preserves all dependency effects in the time series (e.g., autocorrelation and heteroskedasticity). This is also the reason why we do not use a bootstrap approach, where daily stock and cash market returns are drawn randomly with replacement. In this way, any serial dependencies in the time series will be destroyed (Politis, 2003).[18] An alternative that avoids this shortcoming is a block-bootstrap approach, where continuous blocks of 250 daily stock and cash market returns are randomly drawn with replacement (Annaert et al., 2009). However, this approach cannot strictly guarantee that each available year consisting of 250 continuous daily stock and cash market return data will be resampled. Similarly, some years could can be drawn more often than other ones. In contrast, our rolling windows simulation methodology ensures that each window (consisting of 250 daily stock and cash market returns) is exactly considered once.

Table 5
Distributional characteristics of German stock market returns.

	Value	Q-statistic
Panel A: Autocorrelation		
AC (1)	−0.008	Q (1): 0.55
AC (2)	−0.029	Q (2): 7.25*
AC (3)	−0.018	Q (3): 9.67*
AC (4)	0.030	Q (4): 16.47**
AC (5)	−0.010	Q (5): 17.24**
	Value	$T \cdot R^2$
Panel B: Heteroskedasticity		
ARCH (1)	–	293.03**
ARCH (3)	–	824.04**
	Value	Jarque–Bera statistic
Panel C: Skewness, fat tails, and test for normal distribution		
Skewness	−0.32	–
Kurtosis	10.51	–
Test for normality	–	18457.46**

This table shows the distributional characteristics of German stock market returns. The sample contains continuously compounded DAX returns over the period from January 1980 to October 2009 (including 7778 daily stock market returns).
* The test statistic is significant at the 5% level.
** The test statistic is significant at the 1% level.

[17] The money market rates are average values from the one month Frankfurt interbank rate (middle rate).

[18] This is a particular problem for the money market series. While the cash market rates vary significantly during the sample period, they change only slowly as time elapses. Therefore, drawing 250 daily cash market returns randomly leads to a much higher volatility compared to the volatility of 250 subsequently following cash rates. This artificially generated uncertainty in the risk-free rate leads to biased results for some protection strategies.

Table 6
Comparing historical simulations with Monte Carlo simulations.

	CPPI	TIPP	Stop-loss	Synthetic put	Stock market	Cash	50:50 B&H
Panel A: Historical simulations							
Mean return p.a. (%)	6.18	5.21	9.05	8.12	10.62	5.15	7.89
Volatility p.a. (%)	6.57	3.42	17.80	10.82	24.36	2.50	12.12
Sharpe-ratio	0.16	0.02	0.22	0.27	0.22	0.00	0.23
Mean prospect value (λ = 1.0)	4.70	4.17	5.63	5.72	6.93	4.18	5.59
Mean prospect value (λ = 2.25)	4.65	4.14	4.46	5.11	2.22	4.18	3.68
Cumulative prospect value	4.41	3.44	4.85	4.32	−2.05	3.91	0.95
Panel B: Monte Carlo simulations based on real return and risk parameters							
Mean return p.a. (%)	6.44	5.61	7.21	7.13	10.57	5.14	7.86
Volatility p.a. (%)	8.63	2.86	17.77	13.05	24.28	0.00	12.14
Sharpe-ratio	0.15	0.16	0.12	0.15	0.22	–	0.22
Mean prospect value (λ = 1.0)	4.85	4.50	4.40	4.84	6.73	4.22	5.48
Mean prospect value (λ = 2.25)	4.85	4.50	3.57	4.46	2.50	4.22	3.95
Cumulative prospect value	5.54**	3.98**	5.45**	5.22**	−0.05	4.22	2.25

This table compares the results from historical simulations with Monte Carlo simulations. The historical simulation analysis shown in Panel A is based on the daily returns of the German stock and money markets over the sample period from January 1980 to October 2009. We use 250 subsequently following daily returns on a rolling windows basis (moving the window forward by one day) and implement different investment strategies. This approach delivers 7529 overlapping yearly returns for the portfolio insurance strategies and three benchmark investment strategies (a passive stock market fund, a money market fund (cash), and a balanced buy-and-hold strategy (50:50 B&H) without monthly rebalancing). The Monte Carlo simulations shown in Panel B are based on the historical return and risk parameters. The portfolio insurance strategies are implemented with 10 basis points round-trip transaction costs, a trading filter of 2%, and a protection level of 100%. Statistical hypothesis tests in Panel A are complicated by dependence; the computation of confidence intervals based on a moving block bootstrap is discussed in Section 4.2. The null hypothesis in the paired t-test in Panel B is that the cumulative prospect value of a portfolio insurance strategy is equal to that of the money market investment (which is the benchmark strategy with the highest cumulative prospect value).

* The test statistic is significant at the 5% level.

** The test statistic is significant at the 1% level.

The results of our historical simulations are presented in Panel A of Table 6. They reveal that the cumulative prospect value becomes negative for the passive stock market investment (−2.05), indicating the inferiority of this investment strategy for a prospect theory investor. As in our Monte Carlo simulations, the money market investment delivers the highest cumulative prospect value (3.91) and serves as the benchmark strategy. Our main finding is that the CPPI strategy, the stop-loss strategy, and the synthetic put strategy again dominate the money market investment. However, this result is not observable for the TIPP strategy; once more it provides a cumulative prospect value below the money market investment (3.44 vs. 3.91).

While the differences in cumulative prospect values are large in magnitude, testing for differences in mean is highly nontrivial under dependence (Politis, 2003). Our historical simulations produce prospect values that exhibit very high autocorrelation (due to the strongly overlapping windows) and heteroskedasticity. In order to assess statistical significance, we compute the differences between the prospect values of the different portfolio insurance strategies and the money market investment (as the best benchmark strategy) and apply a moving block bootstrap on these difference series. The moving block bootstrap is a resampling method for assigning measures of accuracy to statistical estimates when the observations are in the form of finite time series of correlated data. It does not require special assumptions and consists of drawing blocks of fixed length randomly with replacement and joining them to get new, full-size pseudo-series (Politis, 2003). We follow the algorithm in Politis et al. (1997) to generate confidence intervals for the null hypothesis that the mean of a difference series is equal to zero.[19] To account for a potential bias in the bootstrap distribution, we use the double bootstrap method (Davison and Hinkley, 1997).[20] We experiment with moving block sizes of $n^{1/3}$, $n^{1/4}$, and $n^{1/5}$, where n is the length of a difference series (Politis et al., 1997). Our results from a moving block-bootstrap indicate that the CPPI strategy and the synthetic put strategy significantly dominate the money market investment in terms of their mean and cumulative prospect values independent of the block size. In contrast, although it exhibits the highest cumulative prospect value, for the stop-loss strategy the double bootstrap confidence intervals indicate that its superiority against the cash investment depends on the choice of the block size (with shorter blocks favouring statistical significance). This observation is intuitive given that the stop-loss strategy exhibits the highest annual volatility.

Overall, the results from the Monte Carlo simulations in Section 4.1 and the historical simulation approach in this section are qualitatively similar. Nevertheless, the individual cumulative prospect values differ. It is unclear whether this effect is driven by the different values for the equity risk premium and the volatility of stock returns or by statistical return properties (e.g., left-skewness and fat tails; see Table 5). In order to get more detailed insights, we conduct another Monte Carlo simulation based on the exact historical return and risk parameters, as shown in Table 4. Panel B of Table 6 displays the results. A comparison of Panel A and Panel B reveals that the cumulative prospect values for the four portfolio insurance strategies as well as all benchmark strategies are higher in the Monte Carlo simulations than in the historical simulation approach. This finding suggests that the time series characteristics of stock market returns adversely impact the cumulative prospect values. However, our main result is not affected by these statistical properties, such as skewness and fat tails. The stop-loss strategy, the synthetic put strategy, and the CPPI strategy still dominate the money market investment in terms of their cumulative prospect values; this is not the case for the TIPP strategy.[21]

[19] For more details, see Politis et al. (1997), p. 293.

[20] In order to save computing time in bootstrapping a bootstrap, we resample 500 series in the first-stage bootstrap. For each first-stage resample, we again resample 200 series in the second-stage bootstrap.

[21] The mean prospect values and the cumulative prospect values of the cash investment are identical in all our Monte Carlo simulations. An explanation is that the cash position is assumed to be truly risk-free (a fixed rate with zero volatility) in our Monte Carlo simulations. Therefore, the weighting scheme does not matter, and hence the calculation of the mean prospect values (with equal weighting of all prospect values) leads to the same result as the cumulative prospect values (with varying weighting factors for the individual prospect values). In contrast, in our historical simulations the time series of the risk-free rate exhibits moderate volatility. Accordingly, the weighting scheme matters, and hence we observe different values for the mean prospect values and the cumulative prospect values of the cash investment.

Table 7
Synthetic put strategy and the impact of volatility estimation errors.

	Synthetic put strategy			Stock market	Cash	50:50 B&H
Volatility estimation (%)	24.36	34.36	14.36	–	–	–
Type of estimation error	Unbiased	Overest.	Underest.			
Mean return p.a. (%)	8.12	6.93	9.54	10.62	5.15	7.89
Volatility p.a. (%)	10.82	7.58	14.98	24.36	2.50	12.12
Sharpe-ratio	0.27	0.23	0.29	0.22	0.00	0.23
Mean prospect value (λ = 1.0)	5.72	5.14	6.33	6.93	4.18	5.59
Mean prospect value (λ = 2.25)	5.11	4.87	4.88	2.22	4.18	3.68
Cumulative prospect value	4.32	4.22	3.94	−2.05	3.91	0.95

This table shows the results from historical simulations for the synthetic put strategy and the impact of volatility estimation errors. The results are based on the daily returns of the German stock and money markets over the sample period from January 1980 to October 2009. We use 250 subsequently following daily returns on a rolling windows basis (moving the window forward by one day) and implement the synthetic put strategy. Our rolling windows simulation approach delivers 7529 overlapping yearly returns for the synthetic put strategy and three benchmark investment strategies (a passive stock market fund, a money market fund (cash), and a balanced buy-and-hold strategy (50:50 B&H) without monthly rebalancing). The benchmark case simulates the synthetic put strategy with the "true" stock market volatility of 24.36% (the unbiased volatility estimation). As a comparison, we also simulate the synthetic put strategy with an overestimated volatility of 34.36% (+10 percentage points) and an underestimated volatility of 14.36% (−10 percentage points). The synthetic put strategy is implemented with 10 basis points round-trip transaction costs, a trading filter of 2%, and a protection level of 100%. As discussed in Section 4.2, statistical hypothesis tests are complicated by dependence.

Potential estimation errors are another issue to be considered. In particular, the future stock market volatility is unknown and must be estimated for the synthetic put strategy. Rendleman and O'Brien (1990) document that an "underestimated" volatility leads to an "underprotection" of the synthetic put strategy; in contrast, an "overprotection" due to an "overestimated" volatility causes a reduction in the return potential. The impact of a misestimated volatility on the cumulative prospect values is unclear. In order to get more detailed insight, we repeat our historical simulations for the synthetic put strategy. As in the benchmark case, we simulate the synthetic put strategy with the "true" stock market volatility of 24.36% (the unbiased volatility estimation). As a comparison, we simulate the synthetic put strategy with an overestimated volatility of 34.36% (+10 percentage points) as well as an underestimated volatility of 14.36% (−10 percentage points). Table 7 displays the simulation results. As one would expect, an overestimated volatility leads to a reduced return potential, with a mean annual return of only 6.93% compared to 8.12% in the base case. An underestimated volatility is characterized by higher strategy risk, as indicated by a volatility of 14.98% compared to 10.82% in the base case. An overestimation of the volatility by +10 percentage points only leads to a moderate reduction of the cumulative prospect value (from 4.32 to 4.22). In contrast, an underestimation of the volatility by −10 percentage points leads to a cumulative prospect value of only 3.94. This value is almost identical to the cumulative prospect value of the money market investment (3.91), but it is still higher than the corresponding values of the passive stock market investment (−2.05) and the balanced strategy (0.95). These preliminary findings seem to suggest that volatility misestimation does not exert a large negative impact on the cumulative prospect values.

An obvious limitation of our attempt to incorporate estimation errors is that we assume a constant volatility and a constant estimation error over the entire simulation period. In practice, volatility can change dramatically over time, and hence investors will take this time-variation into account when implementing protection strategies. Therefore, we refine our backtest and incorporate time-varying volatility estimates in the same way as described in Annaert et al. (2009). Specifically, for each simulation year we use the prior 250 daily continuously compounded stock market returns in order to estimate the realized stock market volatility. This rolling volatility is used as a proxy for the expected stock market volatility in the synthetic put strategy. The simulation results shown in Appendix B support our conclusions. A synthetic put strategy implemented with volatility estimates from rolling windows prior to the portfolio formation date dominates all other benchmark strategies in terms of its cumulative prospect value (e.g., a cumulative prospect value of 4.22 for the synthetic put strategy compared to 3.81 for the money market investment). Overall, our results suggest that errors in the estimation of stock return volatility do not have a large negative impact on the cumulative prospect value of a synthetic put strategy.

5. Practical issues: the design of portfolio insurance products

In this section, we analyze the question how portfolio insurance products should be designed and structured in order to meet a prospect theory investor's preferences as accurately as possible (Shefrin and Statman, 1993; Breuer and Perst, 2007; Breuer et al., 2009). An important feature of a portfolio insurance strategy is the optimal protection level (e.g., a protection level of 100%, 95% or 90% of the wealth). Moreover, within a CPPI or a TIPP strategy the multiplier m – the parameter which determines the aggressiveness of a strategy – needs to be determined. We examine both choices based on using our historical simulation approach.

5.1. Choosing the level of protection

While the 95% and the 90% protection variants of a portfolio insurance strategy only provide a reduced downside protection compared to the 100% protection base case version, one would expect higher returns from these strategies. Although this risk-return trade-off may be appropriate in a standard asset pricing context, it is not obvious ex ante if the higher return potential of a protection strategy with a lower floor can offset the reduced downside protection for a prospect theory investor. We run additional historical simulations, in which we analyze the portfolio insurance strategies with protection levels of 95% and 90%. The results are shown in Table 8. The corresponding values for a protection level of 100% are reported in Panel A of Table 6. As one would expect, the mean return of all protection strategies increases when the floor is reduced. For example, the mean return of the CPPI strategy increases from 6.18% to 8.01% per year when the protection level is reduced from 100% to 95%. A further reduction to a protection level of 90% induces an additional increase of the mean return to 9.46% per year. At the same time, one observes an increase in volatility when the protection level is reduced. For the CPPI strategy, the annual volatility increases from 6.57% (with a protection level of 100%) to 17.10% (with a protection level of 90%). Most important, however, a lower protection level

Table 8
Portfolio insurance strategies and the impact of the protection level.

	CPPI	TIPP	Stop-loss	Synthetic put	Stock market	Cash	50:50 B&H
Panel A: Protection level = 95%							
Mean return p.a. (%)	8.01	6.16	10.72	9.30	10.62	5.15	7.89
Volatility p.a. (%)	12.36	6.95	20.30	15.31	24.36	2.50	12.12
Sharpe-ratio	0.23	0.15	0.27	0.27	0.22	0.00	0.23
Mean prospect value (λ = 1.0)	5.50	4.57	6.65	6.19	6.93	4.18	5.59
Mean prospect value (λ = 2.25)	4.40	3.95	3.54	4.23	2.22	4.18	3.68
Cumulative prospect value	3.95	2.77	3.25	3.11	−2.05	3.91	0.95
Panel B: Protection level = 90%							
Mean return p.a. (%)	9.46	7.21	11.04	9.78	10.62	5.15	7.89
Volatility p.a. (%)	17.10	11.14	21.37	18.36	24.37	2.50	12.12
Sharpe-ratio	0.25	0.18	0.28	0.25	0.22	0.00	0.23
Mean prospect value (λ = 1.0)	6.16	5.04	6.99	6.40	6.93	4.18	5.59
Mean prospect value (λ = 2.25)	3.83	3.49	3.22	3.35	2.22	4.18	3.68
Cumulative prospect value	3.11	2.10	2.14	1.75	−2.05	3.91	0.95

This table shows the results from historical simulations for portfolio insurance strategies and the impact of different protection levels. Rather than full (100%) protection, Panel A uses a floor of 95% and Panel B a floor of 90%. The results are based on the daily returns of the German stock and money markets over the sample period from January 1980 to October 2009. We use 250 subsequently following daily returns on a rolling windows basis (moving the window forward by one day) and implement the different investment strategies. Our rolling windows simulation approach delivers 7529 overlapping yearly returns for the portfolio insurance strategies and three benchmark investment strategies (a passive stock market fund, a money market fund (cash), and a balanced buy-and-hold strategy (50:50 B&H) without monthly rebalancing). The portfolio insurance strategies are implemented with 10 basis points round-trip transaction costs, a trading filter of 2%, and a protection level of 100%. As discussed in Section 4.2, statistical hypothesis tests are complicated by dependence.

Table 9
CPPI strategy and the impact of the multiplier.

	CPPI m = 3	CPPI m = 5	CPPI m = 7	CPPI m = 10	Stock market	Cash	50:50 B&H
Mean return p.a. (%)	5.65	6.18	6.83	7.39	10.62	5.15	7.89
Volatility p.a. (%)	3.99	6.57	9.42	12.30	24.36	2.50	12.12
Sharpe-ratio	0.13	0.16	0.18	0.18	0.22	0.00	0.23
Mean prospect value (λ = 1.0)	4.46	4.70	4.97	5.12	6.93	4.18	5.59
Mean prospect value (λ = 2.25)	4.44	4.65	4.88	4.87	2.22	4.18	3.68
Cumulative prospect value	3.86	4.41	4.97	5.18	−2.05	3.91	0.95

This table shows the results from historical simulations for the CPPI strategy and the impact of the multiplier (m). The results are based on the daily returns of the German stock and money markets over the sample period from January 1980 to October 2009. We use 250 subsequently following daily returns on a rolling windows basis (moving the window forward by one day) and implement the CPPI strategy. Our rolling windows simulation approach delivers 7529 overlapping yearly returns for the CPPI strategy and three benchmark investment strategies (a passive stock market fund, a money market fund (cash), and a balanced buy-and-hold strategy (50:50 B&H) without monthly rebalancing). The multiplier determines the level of aggressiveness; it varies between m = 3 and m = 10. The CPPI strategy is implemented with 10 basis points round-trip transaction costs, a trading filter of 2%, and a protection level of 100%. As discussed in Section 4.2, statistical hypothesis tests are complicated by dependence.

leads to a reduction in the cumulative prospect value. This result is observable for all four portfolio insurance strategies. The decrease in the cumulative prospect value is especially pronounced for a protection level of 90%. In this case, all four insurance strategies have lower cumulative prospect values than the money market investment. Presumably, any losses that cannot be recovered within a one year investment horizon are heavily weighted, which then leads to low cumulative prospect values. This observation should be kept in mind when designing financial products with a capital guarantee. For a prospect theory investor it seems more beneficial to structure an investment product that avoids losses as good as possible even if the return potential of such a strategy is dominated by other strategies.

5.2. Choosing the level of aggressiveness in a CPPI strategy

In addition to the floor, the financial engineer must choose the multiplier m for the CPPI strategy.[22] Ceteris paribus, a higher multiplier m leads to higher stock market allocations, and hence it presumably provides should provide a higher return potential. However, a higher multiplier m also increases the gap or overnight risk. Therefore, we analyze the attractiveness of CPPI strategies that are implemented using various multipliers m for a prospect theory investor. We change our base case multiplier m = 5 into m = 3, m = 7, and m = 10 and compare the resulting cumulative prospect values. The results are shown in Table 9. As one would expect, an increasing multiplier m leads to a higher mean return, but also to a higher volatility. Most important, looking at the cumulative prospect values, we observe that an increasing multiplier m leads to a systematic increase in the cumulative prospect values. The CPPI strategy with m = 3 exhibits the lowest cumulative prospect value (3.86), and the CPPI strategy with m = 10 boasts the highest one (5.18); compared to our base case with m = 5, the more aggressive CPPI strategies with m = 7 and m = 10 seem more attractive for a prospect theory investor. However, this is not a general result because a higher multiplier m also causes a higher gap risk. With a multiplier of m = 10 an overnight loss larger than −10% cannot be covered by the CPPI strategy. Although daily losses of this magnitude are extremely rare, a look into the past reveals that they are not impossible (e.g., loss of the German stock market index DAX of −12.80% on 16 October 1989). This observation has another important implication for commercial applications of CPPI strate-

[22] This choice is also necessary for the TIPP strategy. However, our previous results indicate that the CPPI strategy is more preferable for a prospect theory investor than the TIPP strategy. For this reason, we analyze the effects of variations in the multiplier within the CPPI strategy.

gies. In a first step, the maximum tolerable gap or overnight risk must be determined. In a second step, conditional on this gap risk, the highest possible multiplier should be chosen. Put in other words, once the maximum multiplier has been determined according to the acceptable gap or overnight risk, the financial engineer should not select a lower multiplier at least when an investor's preferences are described by cumulative prospect theory.

6. Conclusions

This paper addresses the question why investors often prefer portfolio insurance strategies or guaranteed financial products against other investment strategies. This observation is surprising given that previous studies document that portfolio insurance is hardly optimal in the standard expected utility framework. Our hypothesis is that the popularity of investment strategies with downside protection can be justified in a behavioral finance context. We run Monte Carlo simulations and historical simulations, implement various popular protection strategies together with simple benchmark strategies, and evaluate the outcomes within the framework of cumulative prospect theory. Our results reveal that the standard protection strategies stop-loss, synthetic put, and constant proportion portfolio insurance (CPPI) with a protection level of 100% significantly dominate all benchmark strategies in terms of their cumulative prospect values. However, this result is not observable for the time invariant portfolio protection (TIPP) strategy. In contrast to the other portfolio insurance strategies, the TIPP strategy attempts not only to protect the initial level of wealth, but all interim capital gains as well. This additional protection implies higher opportunity costs in terms of a reduced participation in upward stock market movements. Our simulation results support this hypothesis; the TIPP strategy is dominated by a money market investment in terms of their cumulative prospect values. Our results hold for the Monte Carlo simulation and the historical simulation analyses, and they are robust against several parameter variations (e.g., alternative risk-free rates and alternative probability weighting parameters). Moreover, a practical application of the synthetic put strategy requires estimating the future stock market volatility. Our simulation results reveal that volatility estimation errors do not exert a large negative impact on the cumulative prospect values. Overall, our findings are robust and indicate that the traditional portfolio insurance strategies stop-loss, synthetic put, and CPPI are the preferable investment strategies for prospect theory investors.

Finally, our analysis provides several insights on how a capital guaranteed financial product should be designed and structured in practice in order to meet an investor's preferences as accurately as possible. In spite of the lower return potential, a higher protection level seems more desirable for prospect theory investors than a lower one. Moreover, a CPPI strategy should be implemented as aggressively as possible, with the multiplier being determined based on the maximum tolerable gap or overnight risk.

Acknowledgements

We thank an anonymous referee for helpful comments and Rainer Schlittgen for providing the R code to compute the double bootstrap confidence intervals.

Appendix A. Cumulative prospect values of alternative stock-money market benchmark strategies

	CPPI	TIPP	Stop-loss	Synthetic put	Cash
Panel A: Expected return = 9%, volatility = 20%					
Average stock market allocation (%)	23	10	39	40	–
Cumulative prospect value	3.18**	3.64**	2.46**	2.42**	3.76
Panel B: Expected return = 11.5%, volatility = 20%					
Average stock market allocation (%)	24	10	43	43	–
Cumulative prospect value	3.64**	3.83**	3.22**	3.22**	3.76
Panel C: Expected return = 9%, volatility = 30%					
Average stock market allocation (%)	21	8	28	30	–
Cumulative prospect value	2.68**	3.57**	2.14**	1.98**	3.76
Panel D: Expected return = 11.5%, volatility = 30%					
Average stock market allocation (%)	23	8	30	32	–
Cumulative prospect value	3.00**	3.71*	2.61**	2.50**	3.76

This table shows the results from our base case Monte Carlo simulations for alternative stock-money market benchmark strategies. Continuously compounded stock market returns are generated using a Geometric Brownian motion. The assumptions for expected returns and standard deviations in Panels A and B are taken from the four stock market scenarios presented in Table 1. The risk-free rate is fixed at 4.5%. We simulate 250 daily stock market returns and construct alternative benchmark strategies with a fixed stock-money market allocation without monthly rebalancing. These mixed strategies consist of a $x\%$ stock market allocation and a $(1-x\%)$ cash position, where the fixed weight x denotes the strategy- and scenario-specific average stock market allocation (shown in the table). We perform 100,000 simulation runs in order to derive the full distribution of prospect values for the additional benchmark strategies and the money market investment (which is the benchmark strategy with the highest cumulative prospect value). The null hypothesis in the paired t-test is that the cumulative prospect value of a mixed stock-money market strategy is equal to that of the money market investment.

* The test statistic is significant at the 5% level.

** The test statistic is significant at the 1% level.

Appendix B. Synthetic put strategy with time-varying volatility

	Synthetic put	Stock market	Cash	50:50 B&H
Volatility estimation	Time-varying	–	–	–
Mean return p.a. (%)	7.96	10.99	4.96	7.98
Volatility p.a. (%)	10.86	24.69	2.33	12.31
Sharpe-ratio	0.28	0.24	0.00	0.25
Mean prospect value (λ = 1.0)	5.61	7.17	4.05	5.64
Mean prospect value (λ = 2.25)	5.00	2.35	4.05	3.66
Cumulative prospect value	4.22	−2.03	3.81	0.90

This table shows the results from historical simulations for the synthetic put strategy with time-varying volatility estimates. The results are based on the daily returns of the German stock and money markets over the sample period from January 1980 to October 2009. We use 250 subsequently following daily returns on a rolling windows basis (moving the window forward by one day) and implement the synthetic put strategy starting in mid December 1980 (the first 250 daily return data are required for volatility estimation). Our rolling windows simulation approach delivers 7279 overlapping yearly returns for the synthetic put strategy and three benchmark investment strategies (a passive stock market fund, a money market fund (cash), and a balanced buy-and-hold strategy (50:50 B&H) without monthly rebalancing). For each simulation year we use the prior 250 daily continuously compounded stock market returns in order to estimate the realized stock market volatility; this rolling volatility is used as a proxy for the expected stock market volatility. The synthetic put strategy is implemented with 10 basis points round-trip transaction costs, a trading filter of 2%, and a protection level of 100%. As discussed in Section 4.2, statistical hypothesis tests are complicated by dependence.

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